

Chapter - 12
Linear Programming
Class – XII
Subject – Maths

Exercise-12.1

1. Maximise $Z = 3x + 4y$

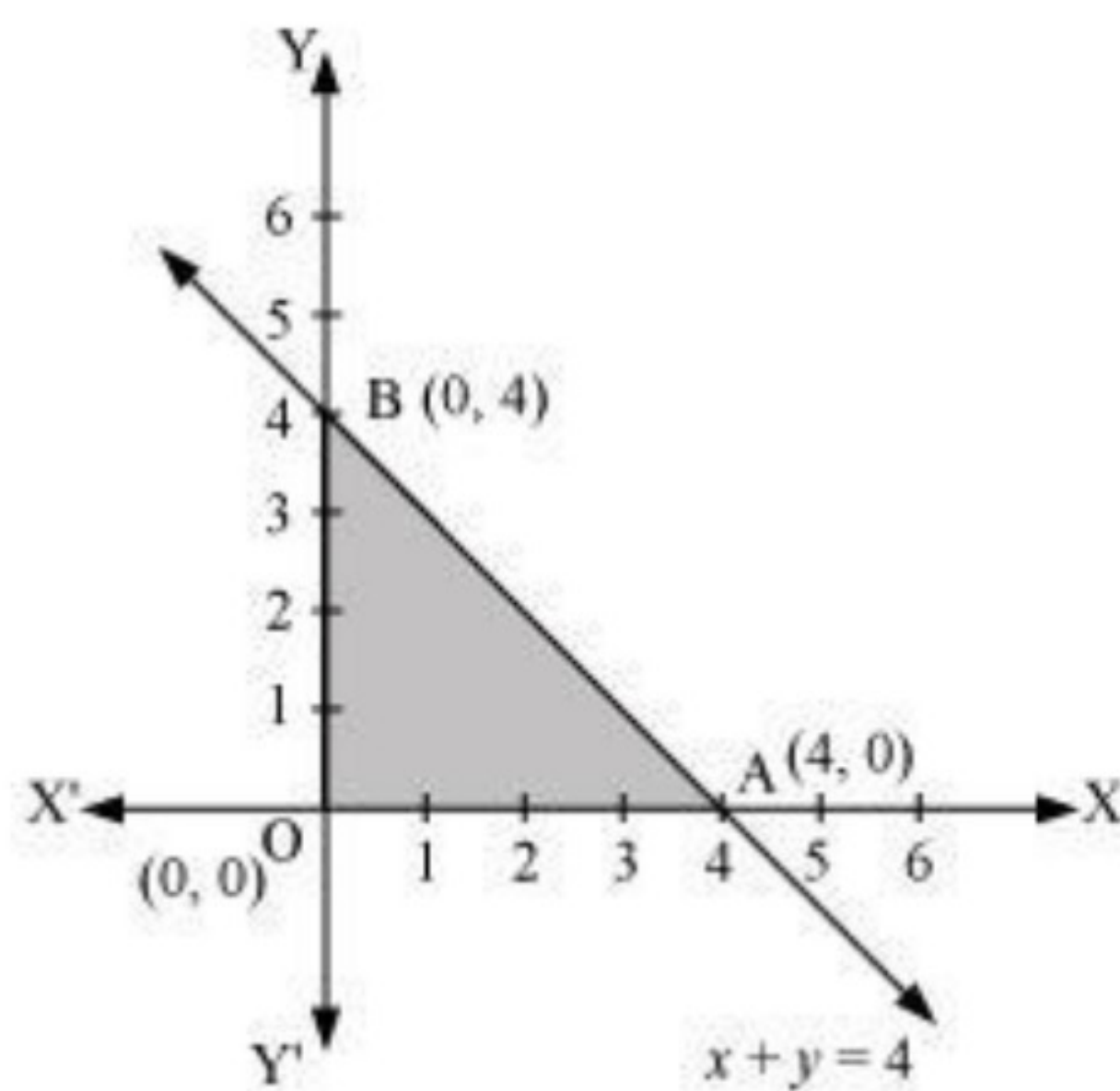
Subject to the constraints: $x + y \leq 4, x \geq 0, y \geq 0$

Sol.

$$x + y \leq 4$$

$$x \geq 0$$

$$y \geq 0$$



Points of shaded region are:

$$O = (0, 0)$$

$$A = (4, 0)$$

$$B = (0, 4)$$

Point	$Z = 3x + 4y$
O(0, 0)	$Z = 3.0 + 4.0 = 0$
A(4, 0)	$Z = 3.4 + 4.0 = 12$
B(0, 4)	$Z = 3.0 + 4.4 = 16$

Therefore, the maximum value of Z is 16 at the point B (0, 4).

2. Minimise $Z = -3x + 4y$

Subject to: $x + 2y \leq 8, 3x + 2y \leq 12, x \geq 0, y \geq 0$

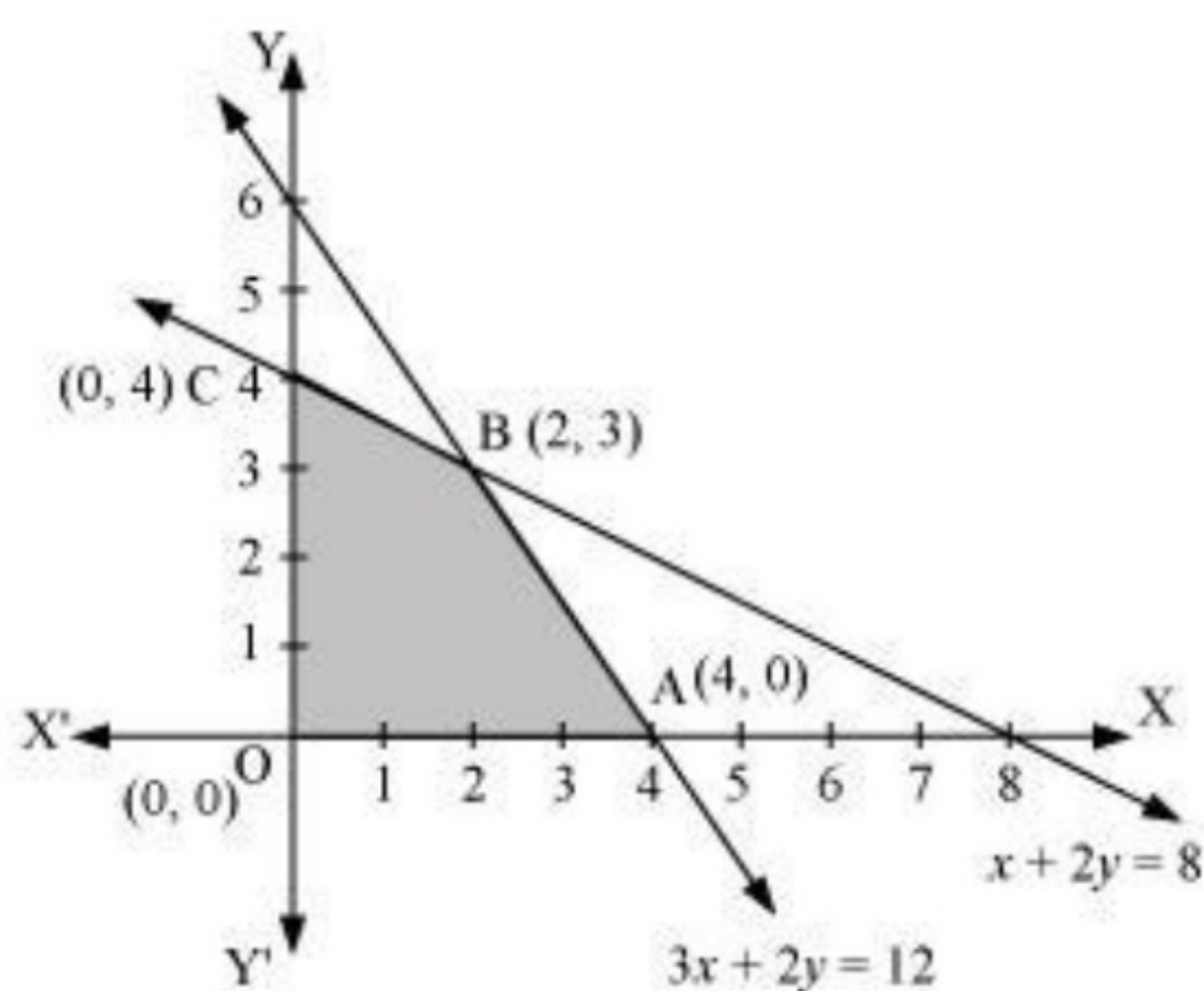
Sol.

$$x + 2y \leq 8$$

$$3x + 2y \leq 12$$

$$x \geq 0$$

$$y \geq 0$$



Points of shaded region are:

$$O = (0, 0)$$

$$A = (4, 0)$$

$$B = (2, 3)$$

$$C = (0, 4)$$

Point	$Z = -3x + 4y$
O(0, 0)	$Z = -3.0 + 4.0 = 0$
A(4, 0)	$Z = -3.4 + 4.0 = -12$
B(2, 3)	$Z = -3.2 + 4.3 = 6$
C(0, 4)	$Z = -3.0 + 4.4 = 16$

Therefore, the minimum value of Z is -12 at the point $(4, 0)$.

3. Maximise $Z = 5x + 3y$

Subject to: $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

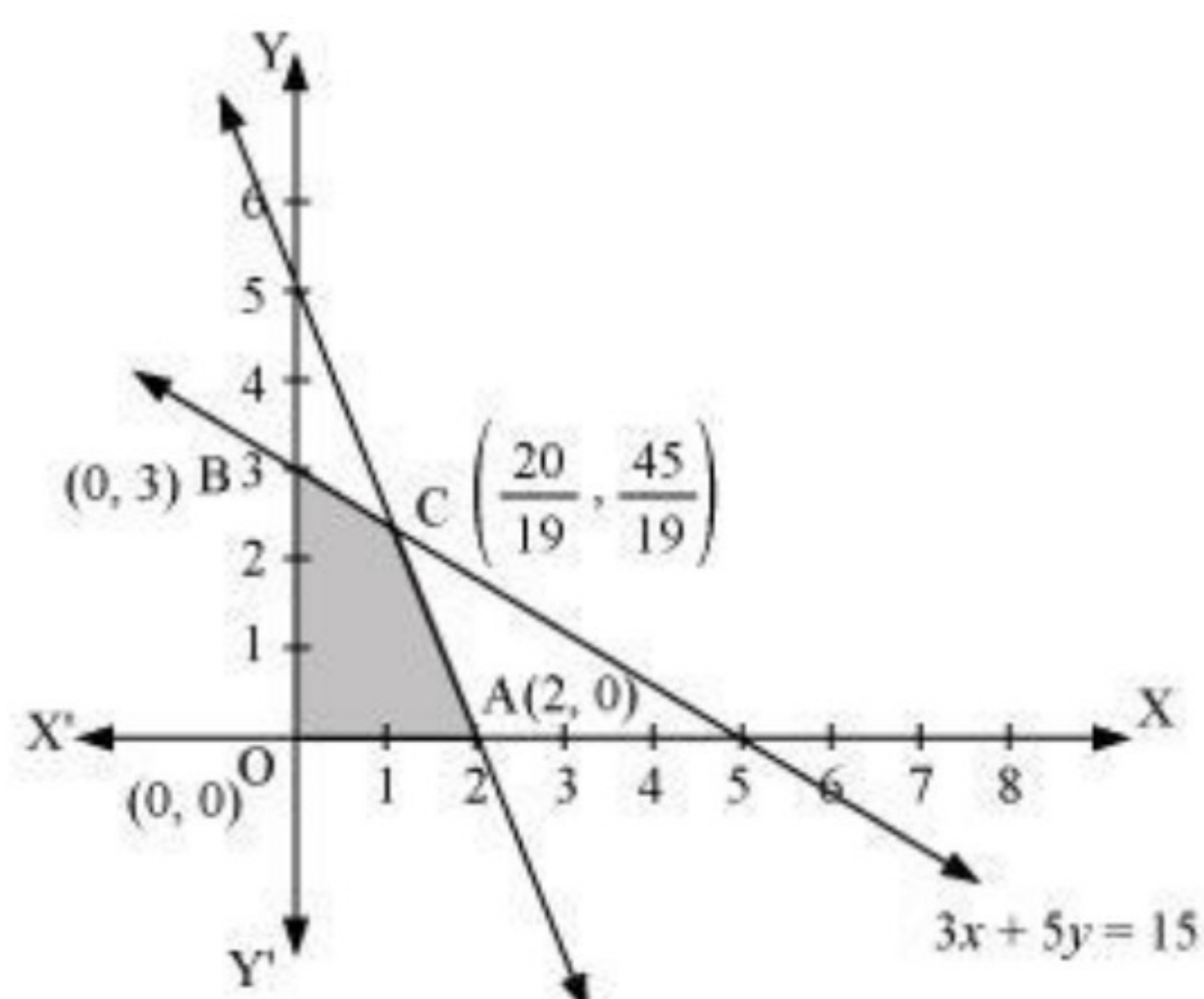
Sol.

$$3x + 5y \leq 15,$$

$$5x + 2y \leq 10$$

$$x \geq 0$$

$$y \geq 0$$



Points of shaded region are:

$$O = (0, 0)$$

$$A = (2, 0)$$

$$B = (0, 3)$$

$$C = \left(\frac{20}{19}, \frac{45}{19}\right)$$

Point	$Z = 5x + 3y$
O(0, 0)	$Z = -5.0 + 3.0 = 0$
A(2, 0)	$Z = 5.2 + 3.0 = 10$
B(0, 3)	$Z = 5.0 + 3.3 = 9$
C $\left(\frac{20}{19}, \frac{45}{19}\right)$	$Z = 5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right) = \frac{235}{19}$

Therefore, the maximum value of Z is $\frac{235}{19}$ at point $C = \left(\frac{20}{19}, \frac{45}{19}\right)$

4. Minimise $Z = 3x + 5y$

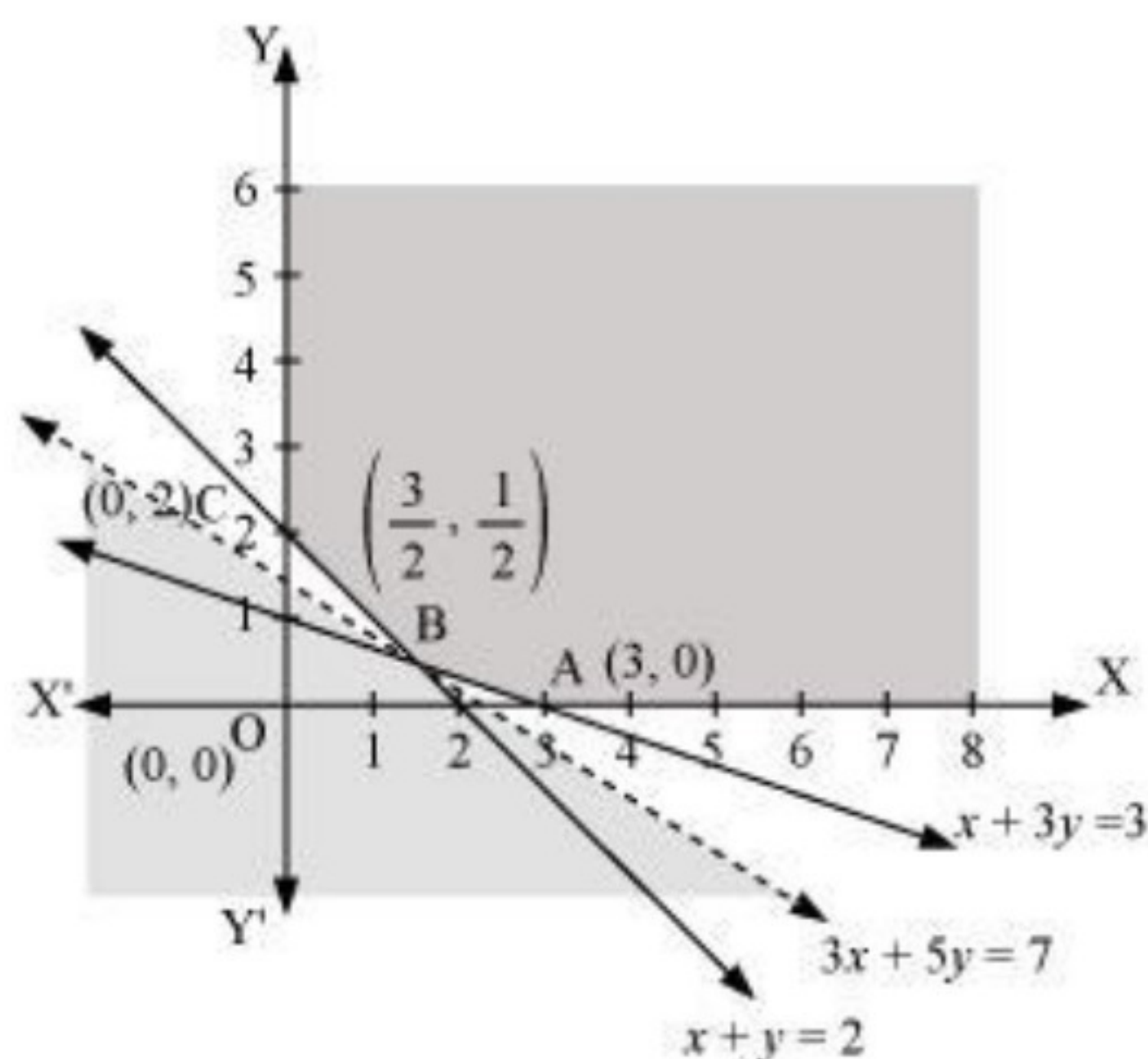
Such that: $x + 3y \geq 3, x + y \geq 2, x, y \geq 0$

Sol.

$$x + 3y \geq 3$$

$$x + y \geq 2$$

$$x, y \geq 0$$



Points of shaded region are:

$$A = (3, 0)$$

$$B = \left(\frac{3}{2}, \frac{1}{2}\right)$$

$$C = (0, 2)$$

Point	$Z = 3x + 5y$
A(3, 0)	$Z = 3.3 + 5.0 = 9$
$B\left(\frac{3}{2}, \frac{1}{2}\right)$	$Z = 3\left(\frac{3}{2}\right) + 5\left(\frac{1}{2}\right) = \frac{14}{2} = 7$
C(0, 2)	$Z = 3.0 + 5.2 = 10$

We can observe that, the feasible region is unbounded.

Thus, 7 is the minimum value of Z at point $B\left(\frac{3}{2}, \frac{1}{2}\right)$

5. Maximise $Z = 3x + 2y$

Subject to: $x + 2y \leq 10$, $3x + y \leq 15$, $x \geq 0$, $y \geq 0$.

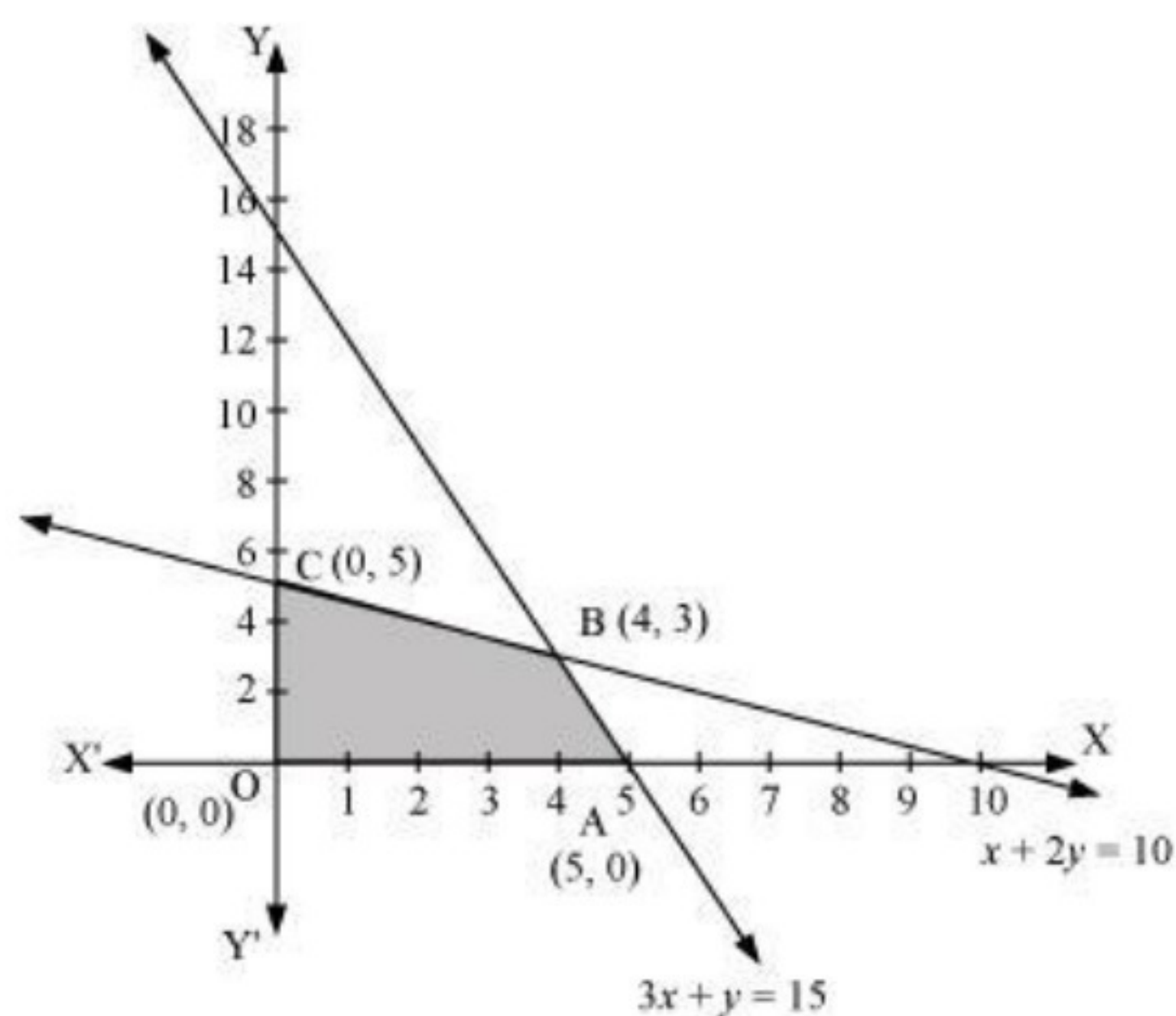
Sol.

$$x + 2y \leq 10$$

$$3x + y \leq 15$$

$$x \geq 0$$

$$y \geq 0$$



Points of shaded region are:

$$A = (5, 0)$$

$$B = (4, 3)$$

$$C = (0, 5)$$

Point	$Z = 3x + 2y$
A(5, 0)	$Z = 3.5 + 2.0 = 15$
B(4, 3)	$Z = 3.4 + 2.3 = 18$
C(0, 5)	$Z = 3.0 + 2.5 = 10$

Thus, the maximum value of Z is 18 at the point (4, 3).

6. Minimise $Z = x + 2y$

Subject to: $2x + y \geq 3$, $x + 2y \geq 6$, $x \geq 0$, $y \geq 0$.

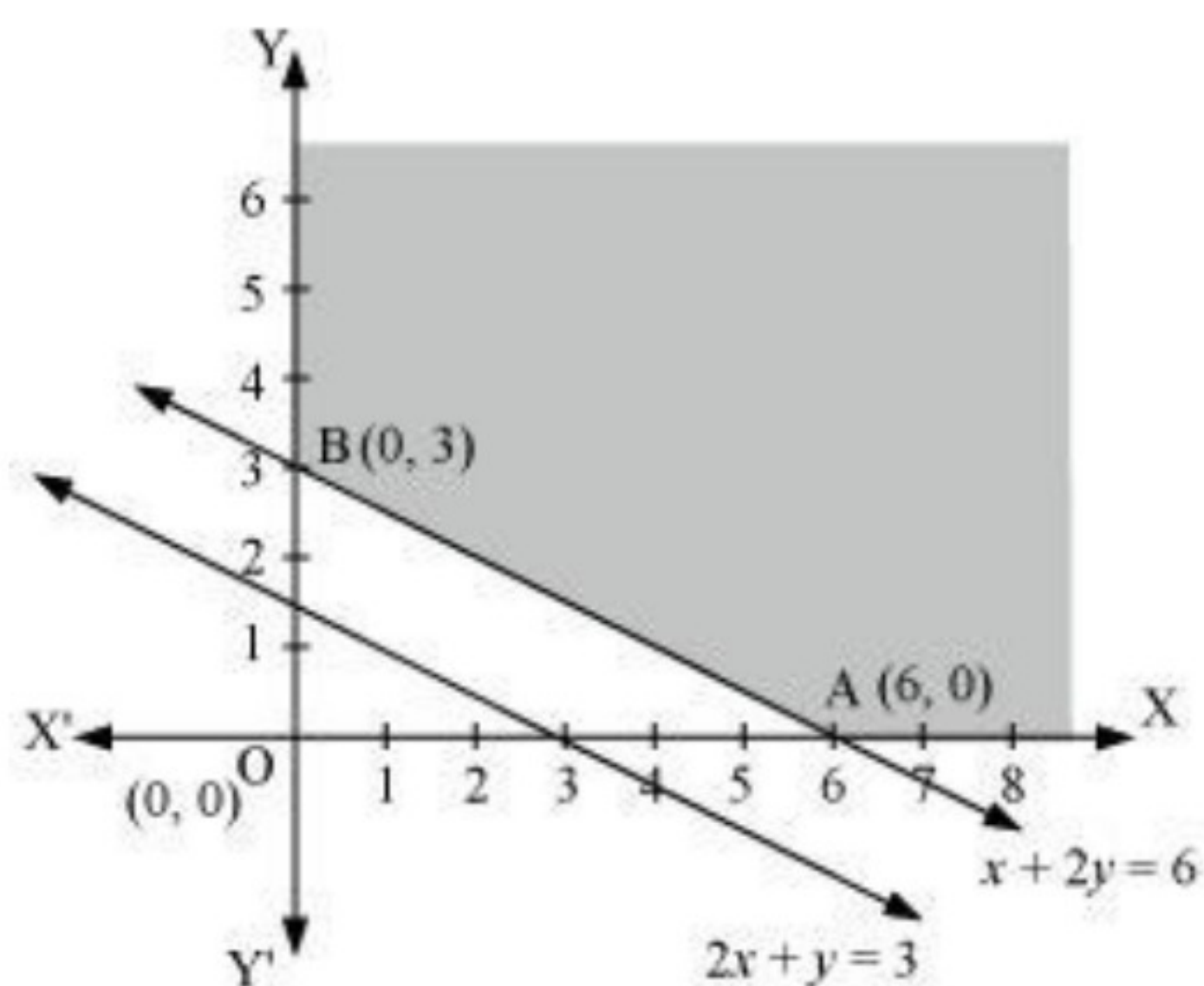
Sol.

$$2x + y \geq 3$$

$$x + 2y \geq 6$$

$$x \geq 0$$

$$y \geq 0$$



Points of shaded region are:

$$A = (6, 0)$$

$$B = (0, 3)$$

Point	$Z = x + 2y$
A(6, 0)	$Z = 6 + 2.0 = 6$
B(0, 3)	$Z = 0 + 2.3 = 6$

Value of Z is same at points A and B.

Thus, the minimum value of Z occurs for more than 2 points.

Therefore, the value of Z is minimum at every point on the line, $x + 2y = 6$

7. Minimise and Maximise $Z = 5x + 10y$

Subject to: $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, $y \geq 0$.

Sol.

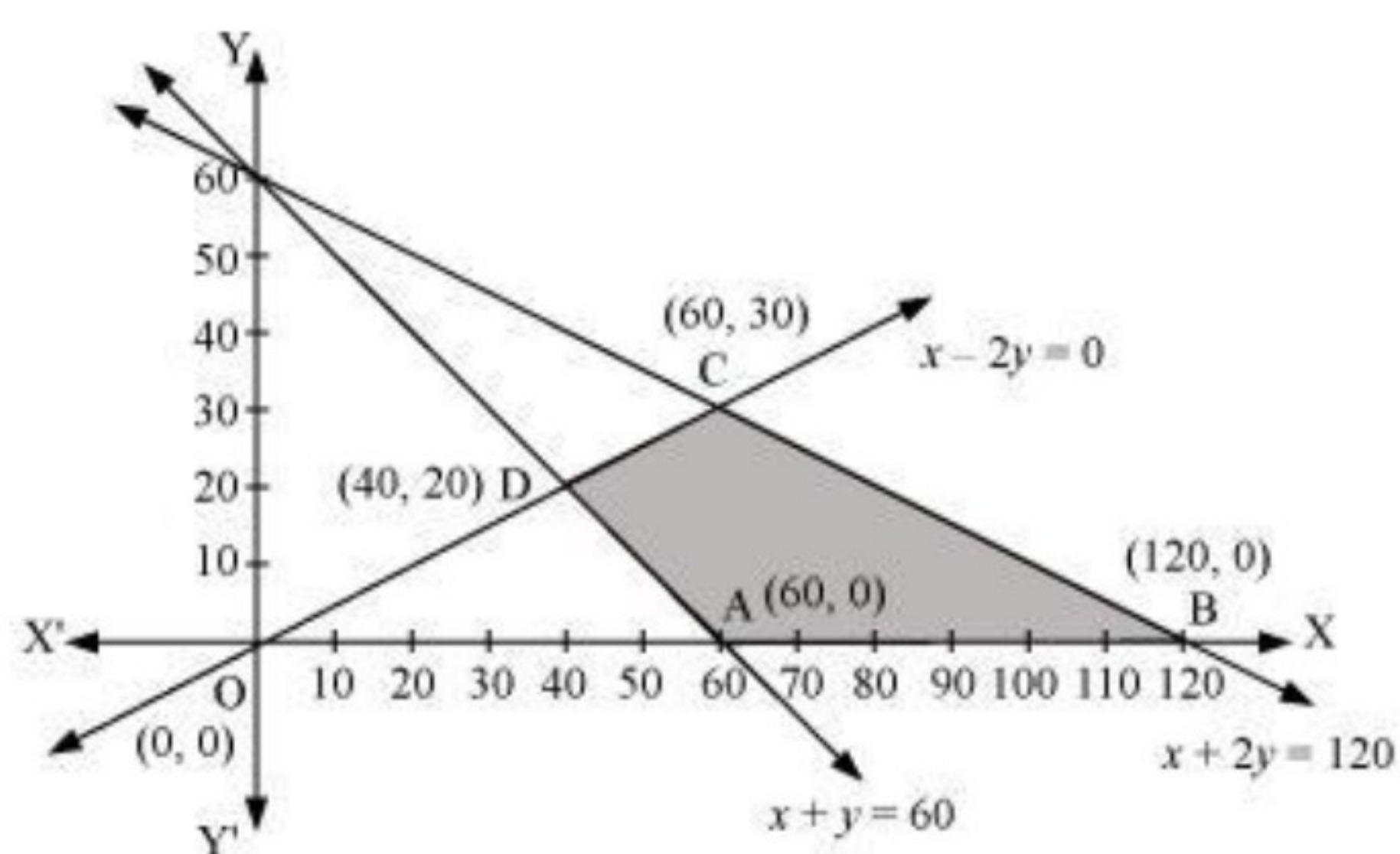
$$x + 2y \leq 120$$

$$x + y \geq 60$$

$$x - 2y \geq 0$$

$$x \geq 0,$$

$$y \geq 0$$



Points of shaded region are:

$$A = (60, 0)$$

$$B = (120, 0)$$

$$C = (60, 30)$$

$$D = (40, 20)$$

Point	$Z = 5x + 10y$
A(60, 0)	$Z = 5.60 + 10.0 = 300$
B(120, 0)	$Z = 5.120 + 10.0 = 600$
C(60, 30)	$Z = 5.60 + 10.30 = 600$
D(40, 20)	$Z = 5.40 + 10.20 = 400$

The minimum value of Z is 300 at (60, 0) and the maximum value of Z is 600 at all the points on the line segment joining (120, 0) and (60, 30).

8. Minimise and Maximise $Z = x + 2y$

Subject to: $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, $y \geq 0$.

Sol.

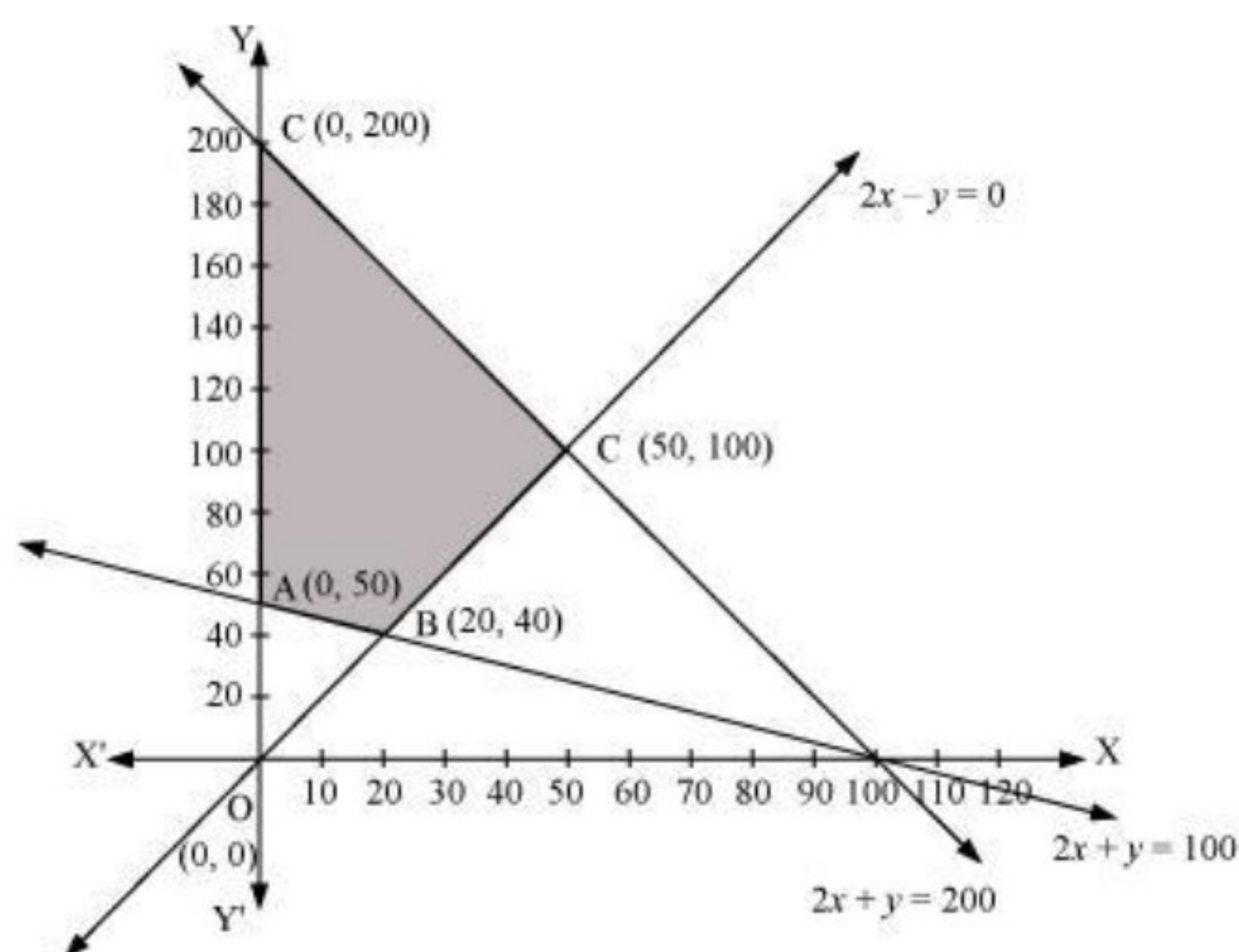
$$x + 2y \geq 100,$$

$$2x - y \leq 0,$$

$$2x + y \leq 200,$$

$$x \geq 0,$$

$$y \geq 0$$



Points of shaded region are:

$$A = (0, 50)$$

$$B = (20, 40)$$

$$C = (50, 100)$$

$$D = (0, 200)$$

Point	$Z = x + 2y$
A(0, 50)	$Z = 0 + 2.50 = 100$
B(20, 40)	$Z = 20 + 2.40 = 100$
C(50, 100)	$Z = 50 + 2.100 = 250$
D(0, 200)	$Z = 0 + 2.200 = 400$

The maximum value of Z is 400 at $(0, 200)$ and the minimum value of Z is 100 at all the points on the line segment joining the points $(0, 50)$ and $(20, 40)$.

9. Maximise $Z = -x + 2y$, subject to the constraints:

$$x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0.$$

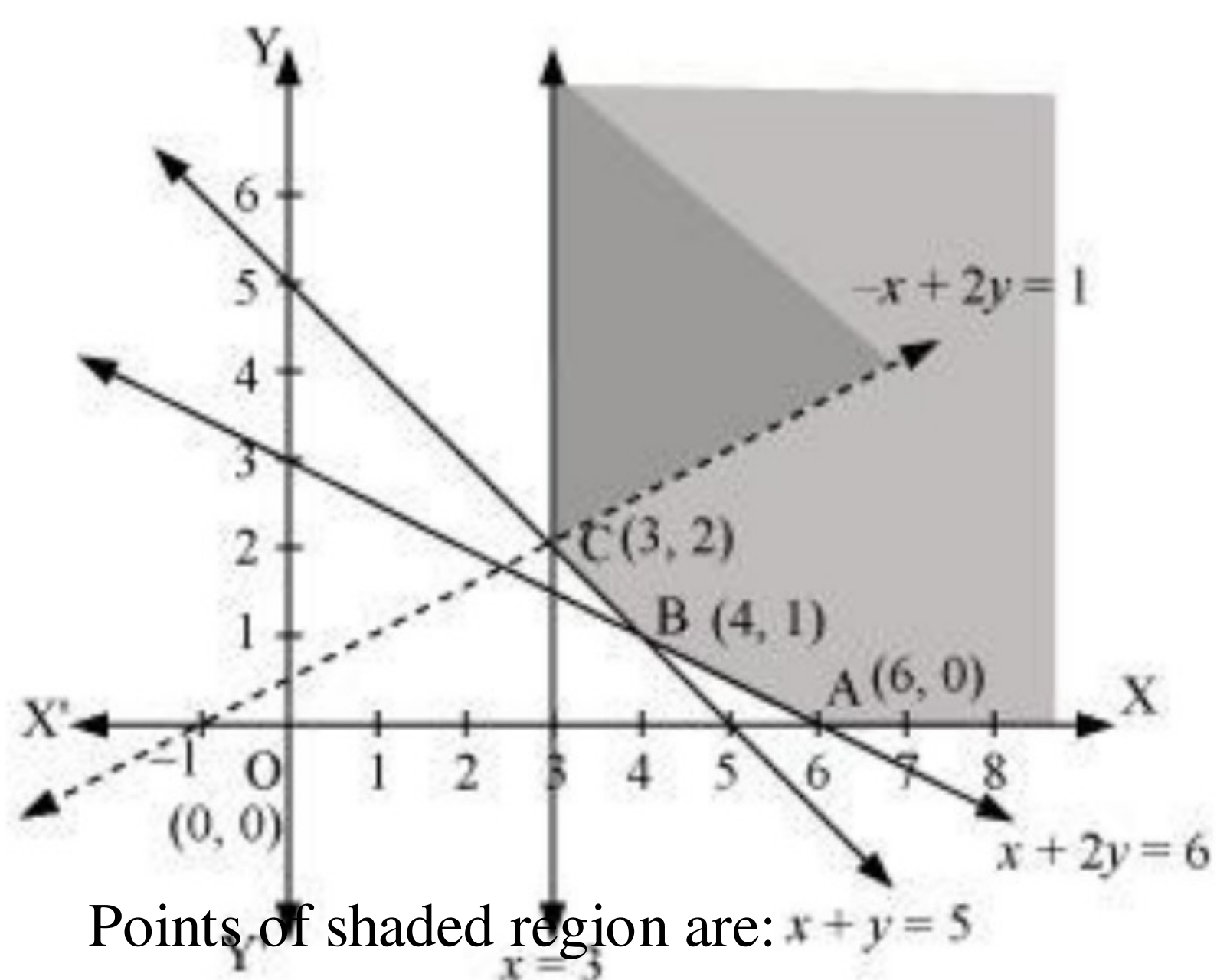
Sol.

$$x \geq 3,$$

$$x + y \geq 5$$

$$x + 2y \geq 6$$

$$y \geq 0$$



$$A = (6, 0)$$

$$B = (4, 1)$$

$$C = (3, 2)$$

Point	$Z = -x + 2y$
A(6, 0)	$Z = -6 + 2.0 = -6$
B(4, 1)	$Z = -4 + 2.1 = -2$
C(3, 2)	$Z = -3 + 2.2 = 1$

It can be seen that the feasible region is unbounded.

Thus, $Z = 1$ is not the maximum value. Z has no maximum value.

10. Maximise $Z = x + y$, subject to:

$$x - y \leq 1,$$

$$-x + y \leq 0,$$

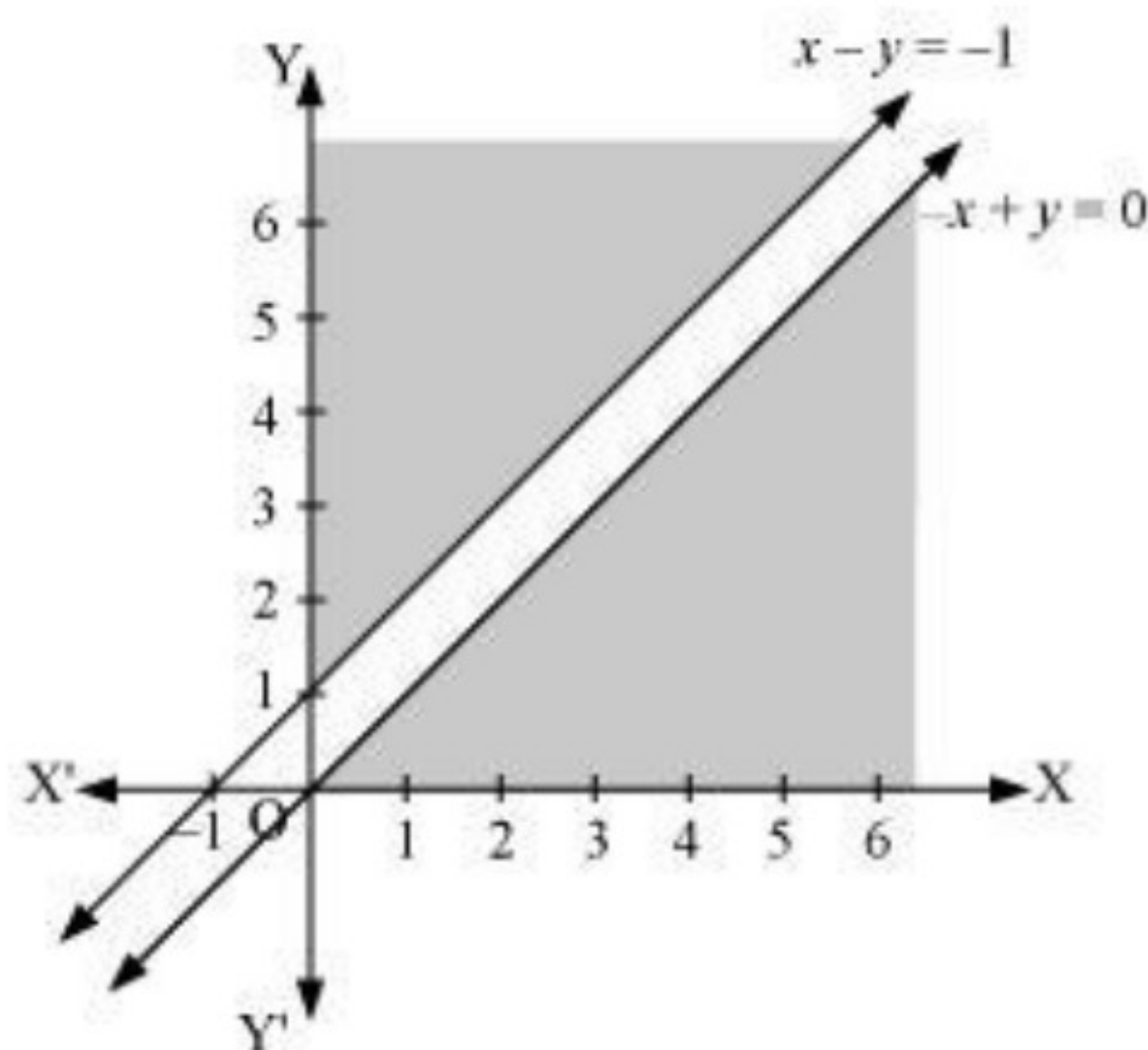
$$x, y \geq 0$$

Sol.

$$x - y \leq 1,$$

$$-x + y \leq 0,$$

$$x, y \geq 0$$



There is no feasible region

Hence, Z has no maximum value.

Exercise-12.2

1. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60/kg and Food Q costs Rs 80/kg. Food P contains 3 units /kg of vitamin A and 5 units /kg of vitamin B while food Q contains 4 units /kg of vitamin A and 2 units /kg of vitamin B. Determine the minimum cost of the mixture?

Sol.

Let the mixture contain = x kg of food P

Amount of food Q = y kg

As per the question,

	Vitamin A (units/kg)	Vitamin B (units/kg)	Cost (Rs/kg)
Food P	3	5	60
Food Q	4	2	80
Requirement (units/kg)	8	11	

$$3x + 4y \geq 8$$

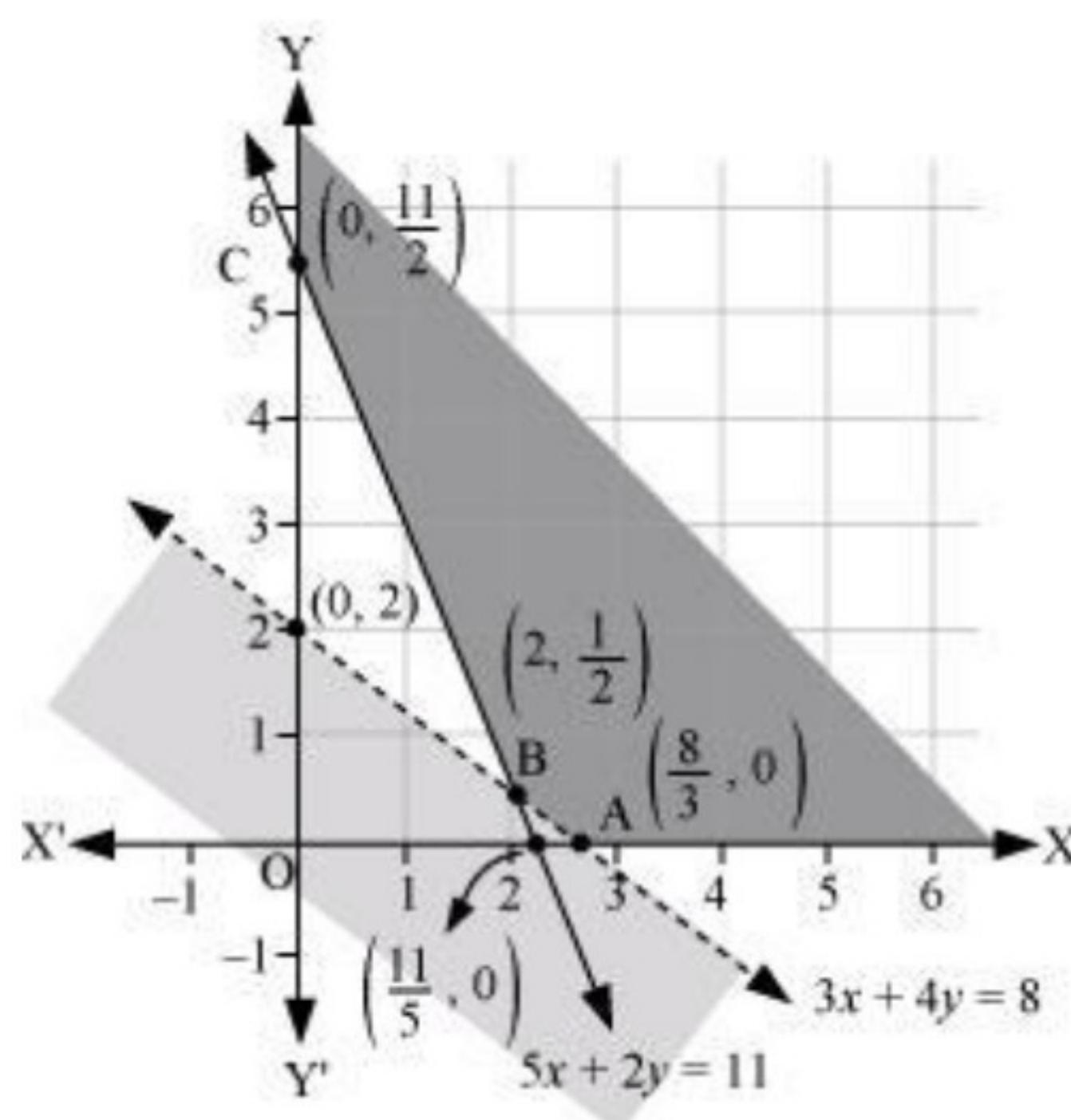
$$5x + 2y \geq 11$$

$$\text{Total cost, } Z = 60x + 80y$$

We have to minimise $Z = 60x + 80y$

$$x, y \geq 0$$

Now,



The corner points of the feasible region are:

$$A = \left(\frac{8}{3}, 0 \right)$$

$$B = \left(2, \frac{1}{2} \right)$$

$$C = \left(0, \frac{11}{2} \right)$$

Point	$Z = 60x + 80y$
$A \left(\frac{8}{3}, 0 \right)$	$Z = 60 \cdot \frac{8}{3} + 80 \cdot 0 = 160$
$B \left(2, \frac{1}{2} \right)$	$Z = 60 \cdot 2 + 80 \cdot \frac{1}{2} = 160$
$C \left(0, \frac{11}{2} \right)$	$Z = 60 \cdot 0 + 80 \cdot \frac{11}{2} = 440$

Therefore, the minimum cost of the mixture will be Rs 160 at the line segment joining the points $\left(\frac{8}{3}, 0 \right)$ and $\left(2, \frac{1}{2} \right)$.

- 2. One kind of cake requires 200g flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes?**

Sol.

Let cakes of first kind = x

Cakes of second kind = y

As per the question,

	Flour (g)	Fat (g)
Cakes of first kind, x	200	25
Cakes of second kind, y	100	50
Availability	5000	1000

$$200x + 100y \leq 5000$$

$$2x + y \leq 50 \dots\dots\dots (1)$$

$$25x + 50y \leq 1000$$

$$x + 2y \leq 40 \dots\dots\dots (2)$$

Total numbers of cakes, $Z = x + y$

We have to maximize $Z = x + y$

Equations are:

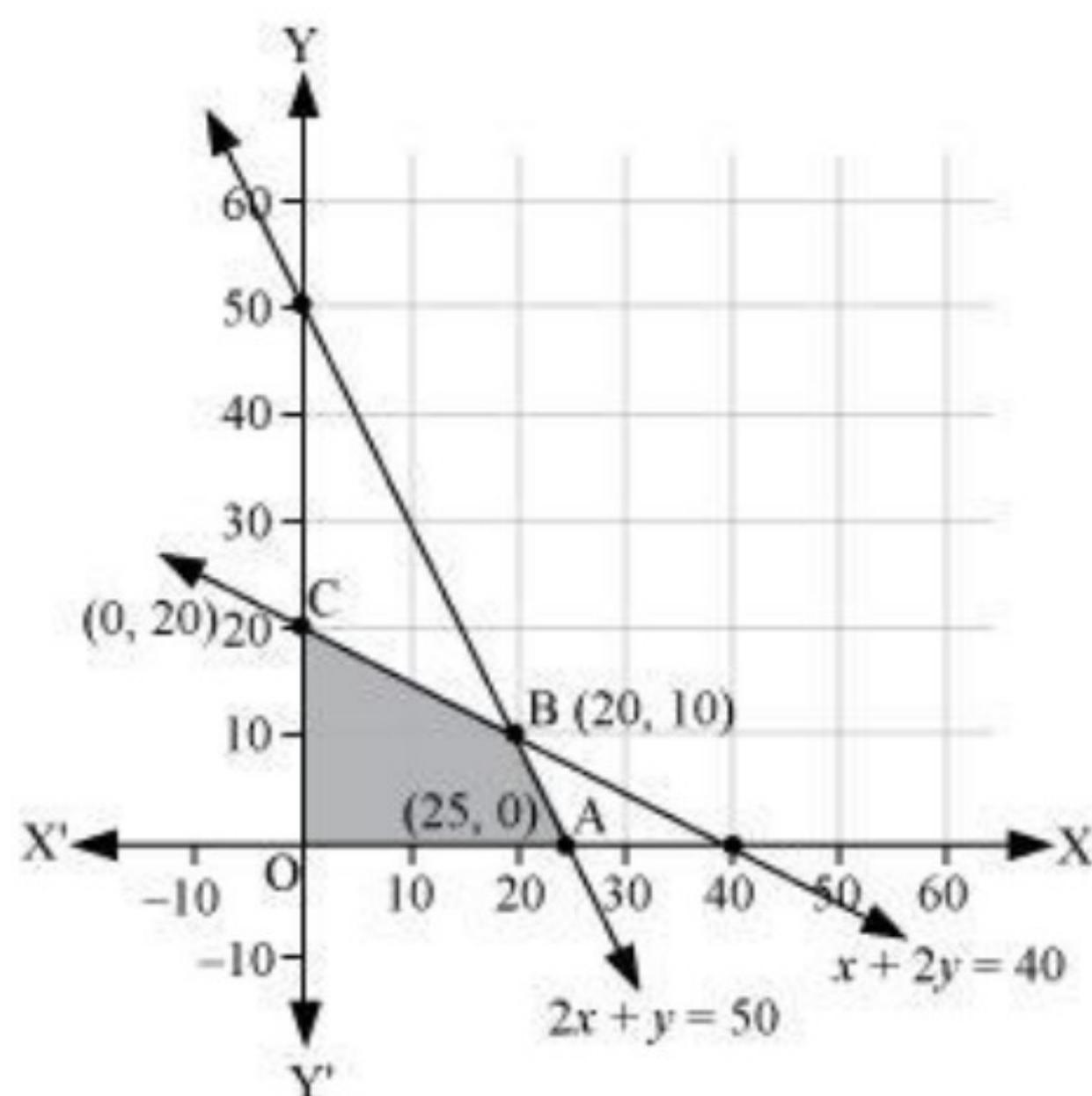
$$2x + y \leq 50$$

$$x + 2y \leq 40$$

$$x \geq 0$$

$$y \geq 0$$

Now,



The corner points are:

$$A = (25, 0)$$

$$B = (20, 10)$$

$$O = (0, 0)$$

$$C = (0, 20)$$

The values of Z are:

Corner point	$Z = x + y$
A(25, 0)	25
B(20, 10)	30
C(0, 20)	20
O(0, 0)	0

Thus, the maximum numbers of cakes that can be made are 30.

- 3. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.**

(ii) What number of rackets and bats must be made if the factory is to work at full capacity?

(ii) If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity.

Sol.

(i) Let the number of rackets = x

The number of bats = y

As per the question,

$$1.5x + 3y = 42$$

$$3x + y = 24$$

On solving these equations, we have

$$x = 4 \text{ and } y = 12$$

Thus, rackets are 4 and bats are 12

(ii)

	Tennis Racket	Cricket Bat	Availability
Machine Time (h)	1.5	3	42
Craftsman's Time (h)	3	1	24

$$1.5x + 3y \leq 42$$

$$3x + y \leq 24$$

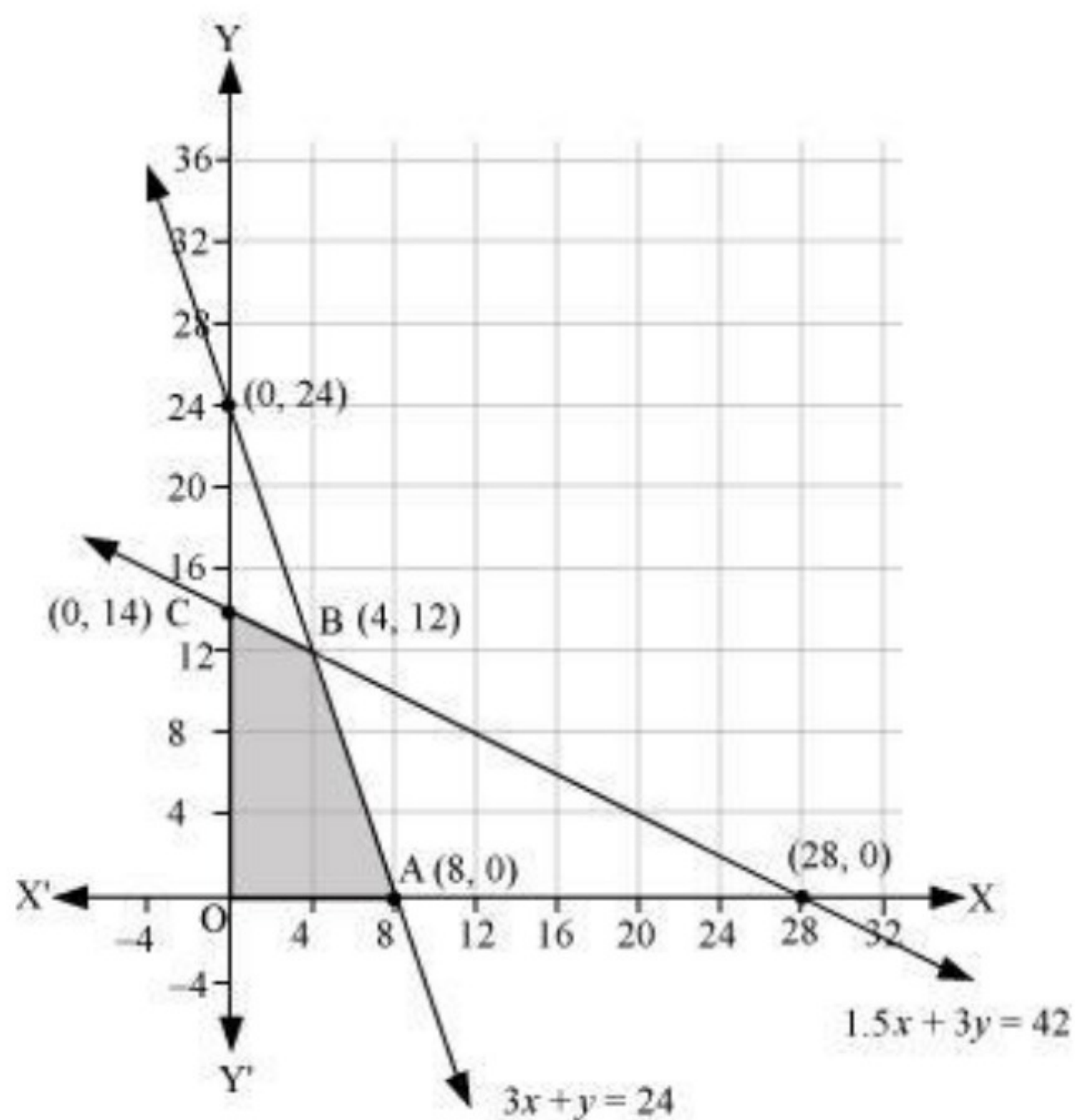
$$x, y \geq 0$$

The profit on a racket = Rs 20

Profit on a bat = Rs 10

$$Z = 20x + 10y$$

Now,



The corner points are:

$$A = (8, 0)$$

$$B = (4, 12)$$

$$C = (0, 14)$$

$$O = (0, 0)$$

Now,

Corner point	$Z = 20x + 10y$
A(8, 0)	160
B(4, 12)	200
C(0, 14)	140
O(0, 0)	0

Thus, the maximum profit of the factory when it works to its full capacity is Rs 200.

- 4. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit, of Rs 17.50 per package on nuts and Rs. 7.00 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 12 hours a day?**

Sol.

Let the package of nuts = x

Packages of bolts = y

As per the question,

	Nuts	Bolts	Availability
Machine A (h)	1	3	12
Machine B (h)	3	1	12

The profit on a package of nuts = Rs 17.50

Profit on a package of bolts = Rs 7

Total profit, $Z = 17.5x + 7y$

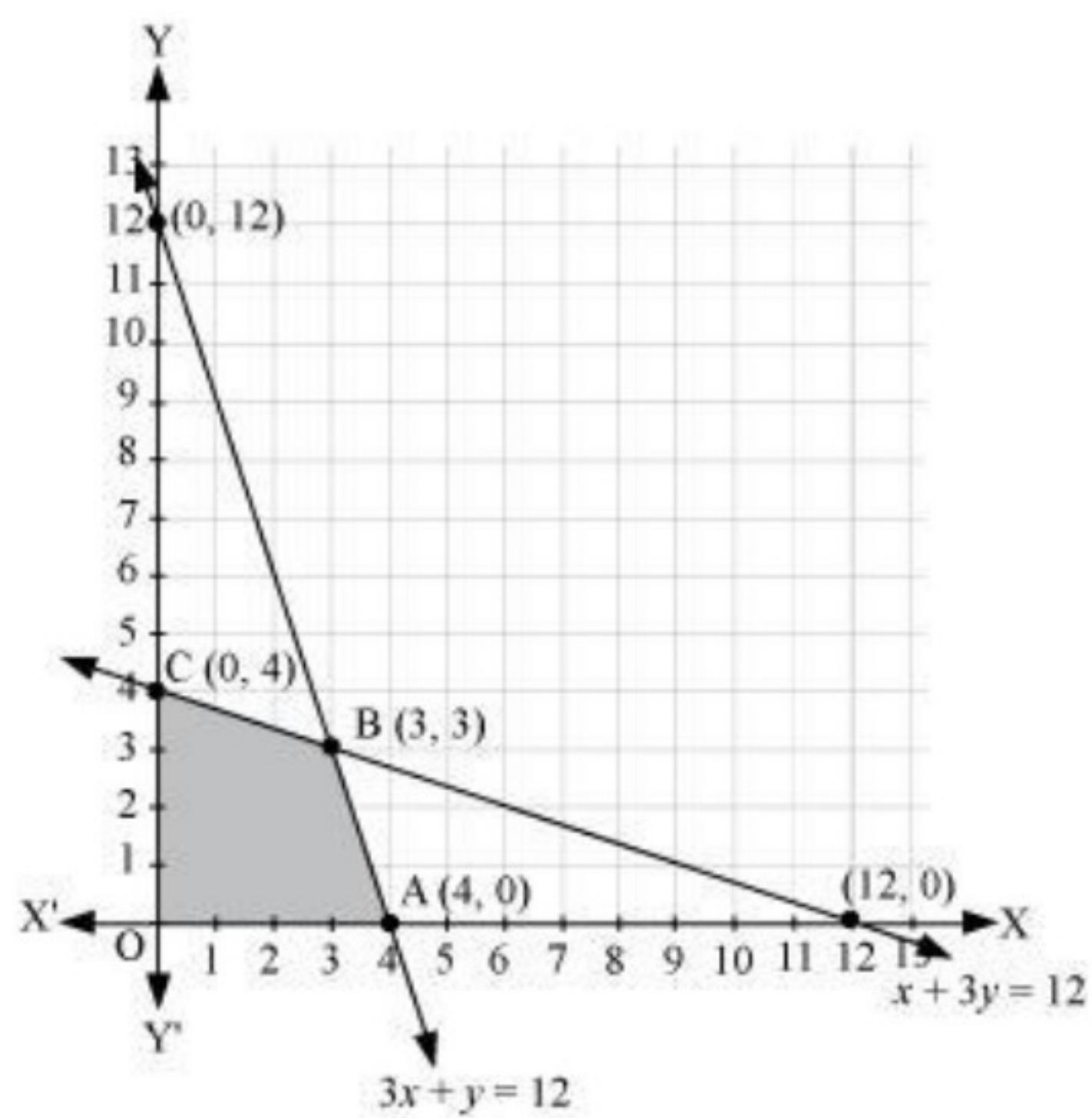
The equations are:

$$x + 3y \leq 12$$

$$3x + y \leq 12$$

$$x, y \geq 0$$

Now,



The corner points are:

$$A = (4, 0)$$

$$B = (3, 3)$$

$$C = (0, 4)$$

Now,

Corner point	$Z = 17.5x + 7y$
O(0, 0)	0
A(4, 0)	70
B(3, 3)	73.5
C(0, 4)	28

The maximum value of Z is Rs 73.50 at (3, 3).

Thus, 3 packages of nuts and 3 packages of bolts should be produced each day to get the maximum profit of Rs 73.50.

- 5. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit.**

Sol.

Let screws of type A = x

Screws of type B = y

As per the question,

	Screw A	Screw B	Availability
Automatic Machine (min)	4	6	$4 \times 60 = 120$
Hand Operated Machine (min)	6	3	$4 \times 60 = 120$

The profit on a package of screws A = Rs 7

Profit on the package of screws B = Rs 10

Total profit, $Z = 7x + 10y$

Now,

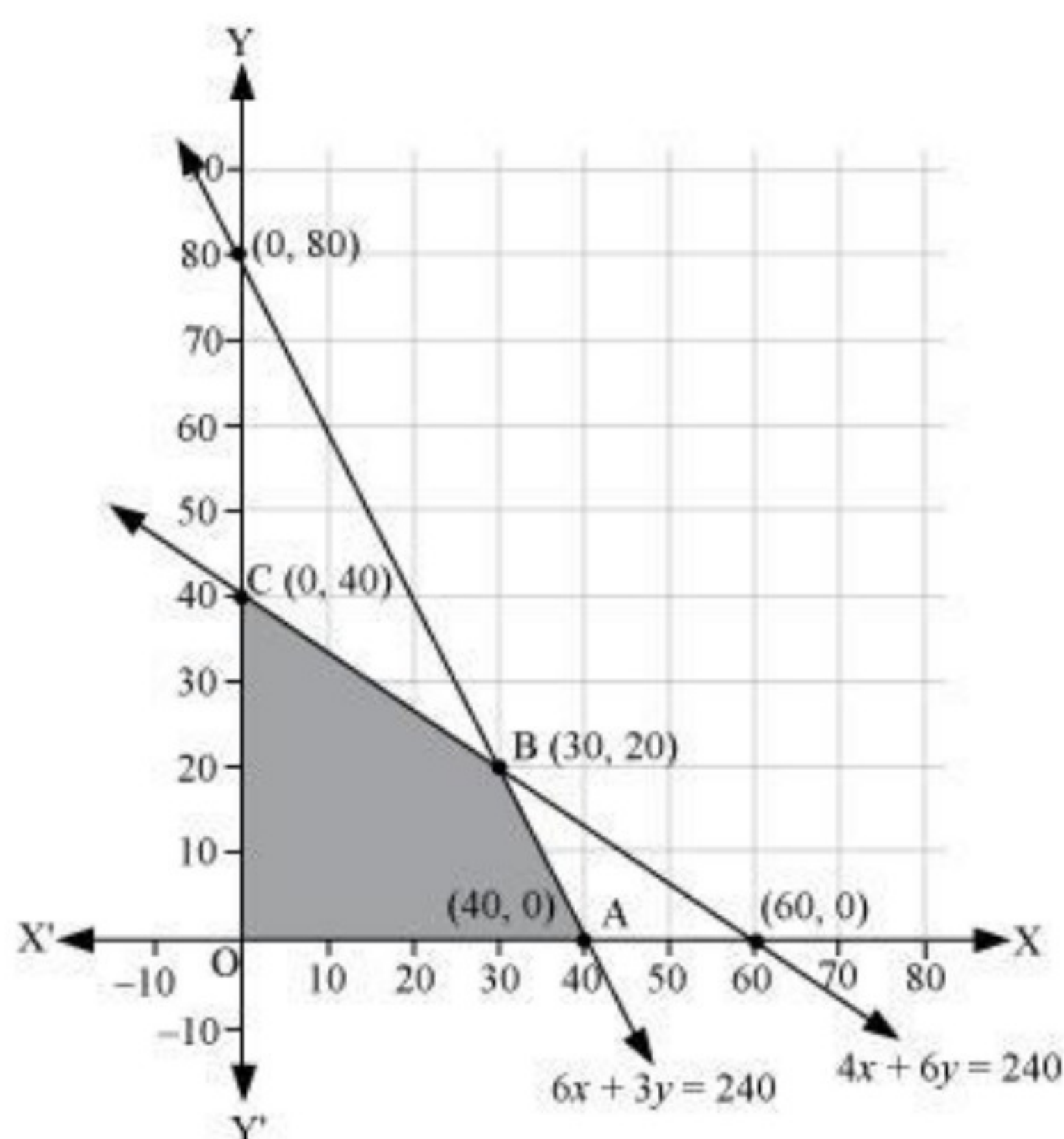
$$4x + 6y \leq 240$$

$$6x + 3y \leq 240$$

$$x \geq 0$$

$$y \geq 0$$

Now,



The corner points are:

$$A = (40, 0)$$

$$B = (30, 20)$$

$$C = (0, 40)$$

The value of Z is:

Corner point	$Z = 7x + 10y$
A(40, 0)	280
B(30, 20)	410
C(0, 40)	400

The maximum value of Z is 410 at (30, 20).

Thus, the factory should produce 30 packages of screws A and 20 packages of screws B to get the maximum profit of Rs 410.

- 6. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs 5 and that from a shade is Rs 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximize his profit?**

Sol.

Let the cottage industry manufacture pedestal lamps = x

Wooden shades = y

As per the question,

	Lamps	Shades	Availability
Grinding/Cutting Machine (h)	2	1	12
Sprayer (h)	3	2	20

The profit on a lamp = Rs 5

Profit on the shades = Rs 3

Total profit, $Z = 5x + 3y$

The equations are:

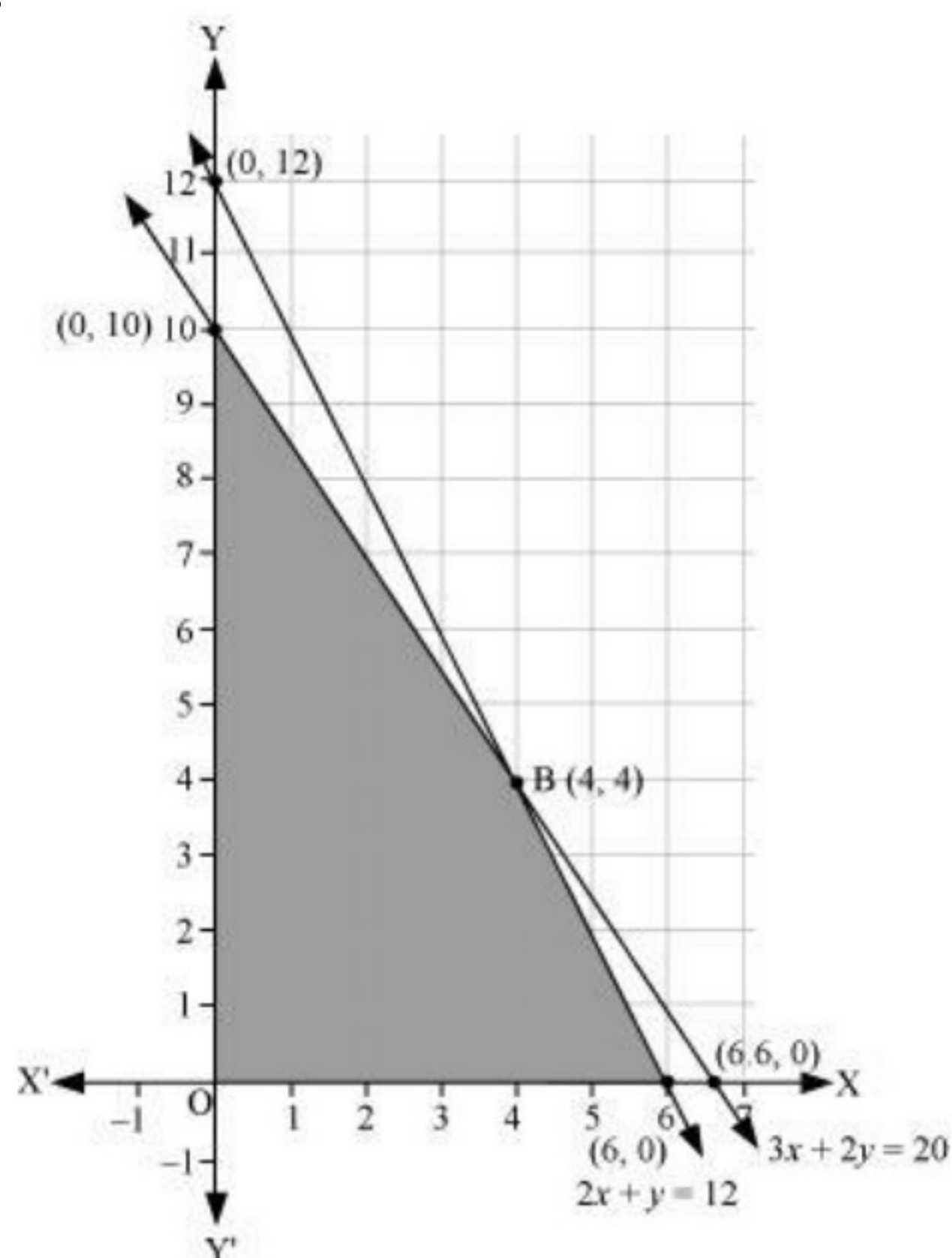
$$2x + y \leq 12$$

$$3x + 2y \leq 20$$

$$x \geq 0$$

$$y \geq 0$$

Now,



The corner points are:

$$A = (6, 0)$$

$$B = (4, 4)$$

$$C = (0, 10)$$

The values of Z are:

Corner point	$Z = 5x + 3y$
A(6, 0)	30
B(4, 4)	32
C(0, 10)	30

The maximum value of Z is 32 at (4, 4).

Thus, the manufacturer should produce 4 pedestal lamps and 4 wooden shades to maximize his profits.

- 7. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours of assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit?**

Sol.

Let souvenirs of type A = x

Souvenirs of type B = y

As per the question,

	Type A	Type B	Availability
Cutting (min)	5	8	$3 \times 60 + 20 = 200$
Assembling (min)	10	8	$4 \times 60 = 240$

The profit on souvenirs of type A = Rs 5

Profit on type B souvenirs = Rs 6

Total profit, $Z = 5x + 6y$

The equations are:

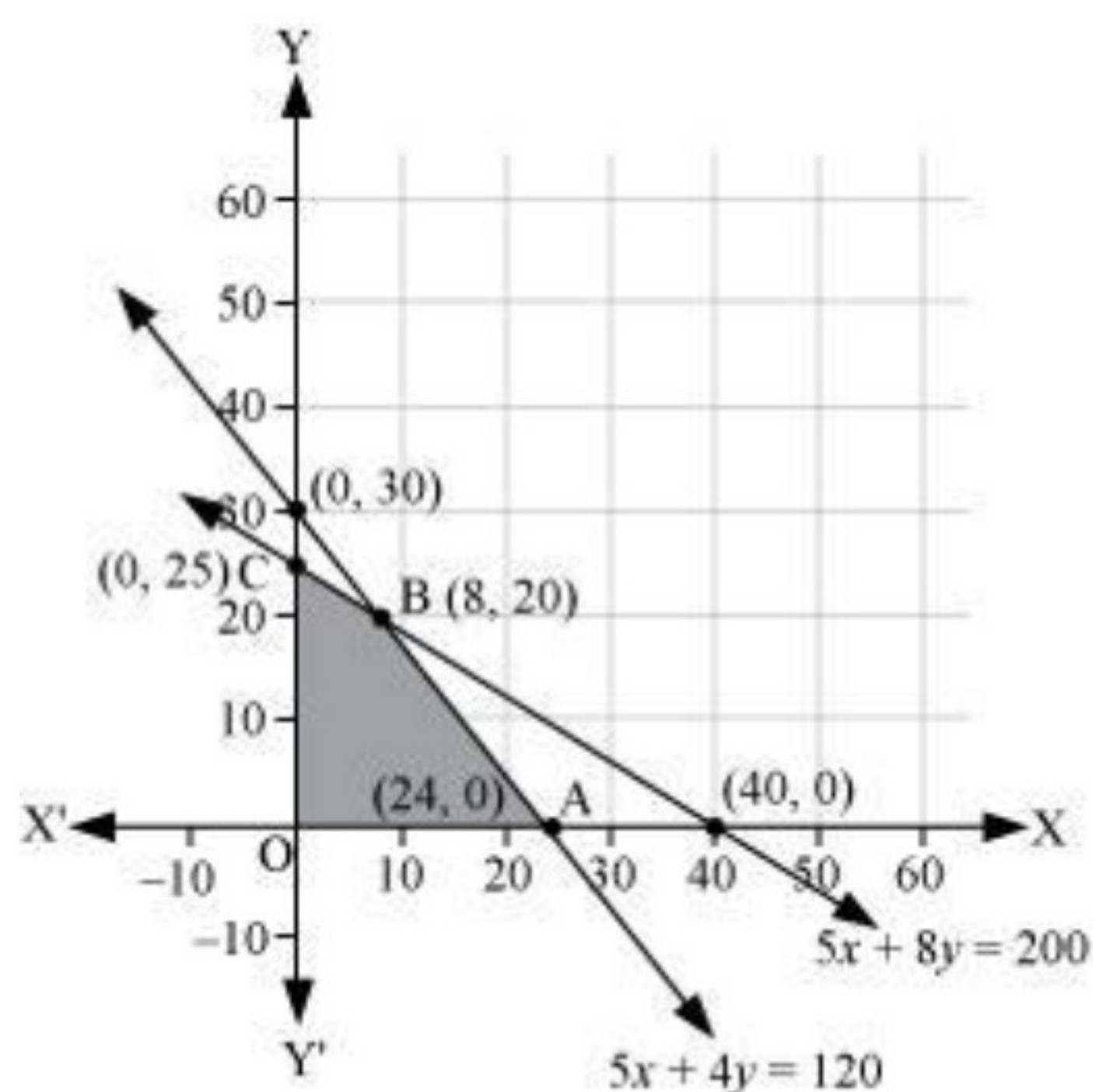
$$5x + 8y \leq 200$$

$$5x + 4y \leq 120$$

$$x \geq 0$$

$$y \geq 0$$

Now,



The corner points are:

$$A = (24, 0)$$

$$B = (8, 20)$$

$$C = (0, 25)$$

The values of Z are:

Corner point	$Z = 5x + 6y$
A(24, 0)	120
B(8, 20)	160
C(0, 25)	150

The maximum value of Z is 200 at (8, 20).

Thus, 8 souvenirs of type A and 20 souvenirs of type B should be produced each day to get the maximum profit of Rs 160.

- 8. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4500 and on portable model is Rs 5000.**

Sol.

Let the merchant stock desktop models = x

Portable models = y

The cost of a desktop model is = Rs 25000

Cost of a portable model = Rs 4000.

Thus,

$$25000x + 40000y \leq 7000000$$

$$5x + 8y \leq 1400$$

The monthly demand of computers will not exceed 250 units.

$$x + y \leq 250$$

The profit on a desktop model = Rs 4500

Profit on a portable model = Rs 5000

$$\text{Total profit, } Z = 4500x + 5000y$$

The equations are:

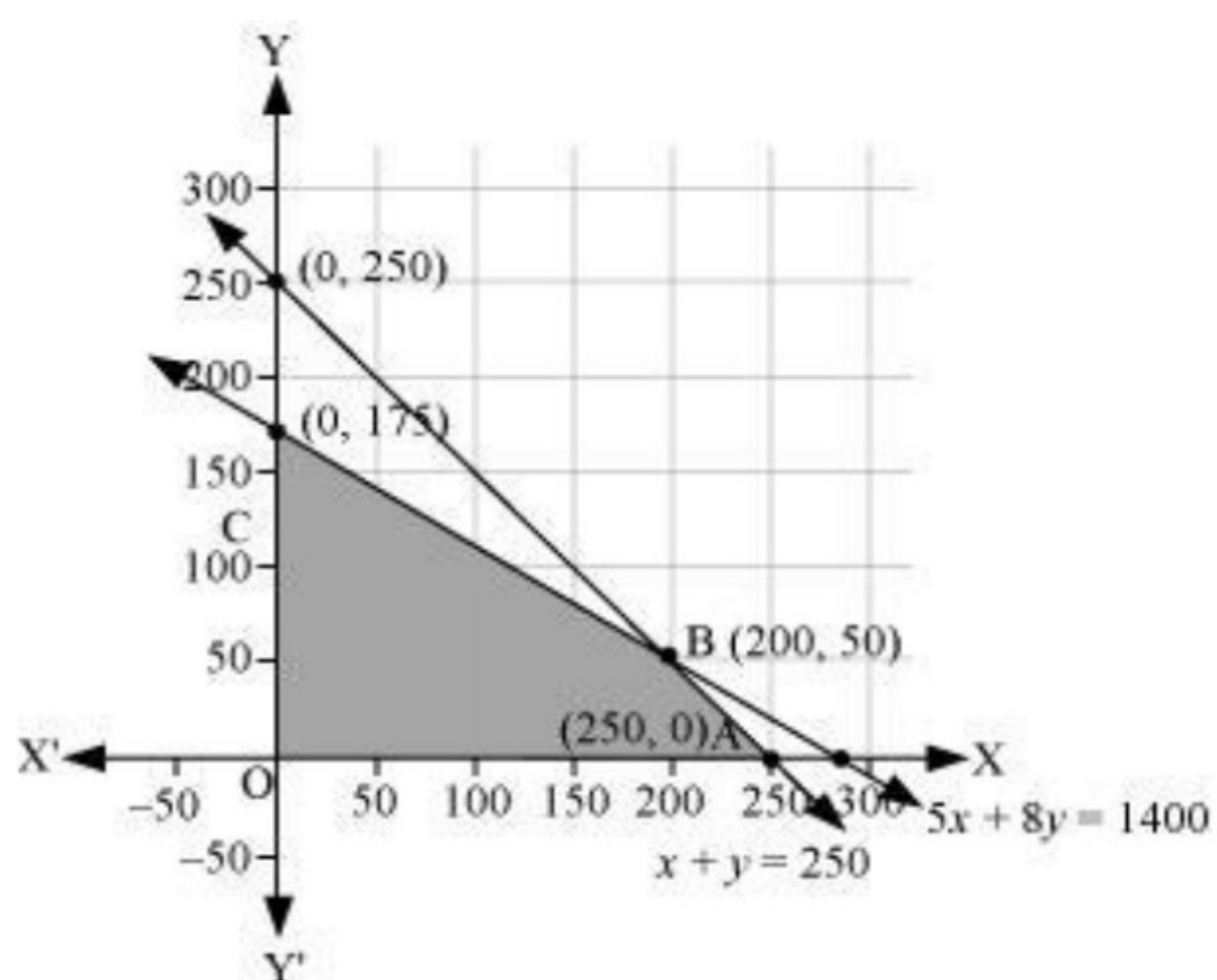
$$5x + 8y \leq 1400$$

$$x + y \leq 250$$

$$x \geq 0$$

$$y \geq 0$$

Now,



The corner points are:

$$A = (250, 0)$$

$$B = (200, 50)$$

$$C = (0, 175)$$

The values of Z are:

Corner point	$Z = 4500x + 5000y$
A(250, 0)	1125000
B(200, 50)	1150000
C(0, 175)	875000

The maximum value of Z is 1150000 at (200, 50).

Thus, the merchant should stock 200 desktop models and 50 portable models to get the maximum profit of Rs 1150000.

- 9. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs 4 per unit food and F_2 costs Rs 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements?**

Sol.

Let the diet contain units of food $F_1 = x$

Units of food $F_2 = y$

	Vitamin A (units)	Mineral (units)	Cost per unit (Rs)
Food F₁ (x)	3	4	4
Food F₂ (y)	6	3	6
Requirement	80	100	

As per the question,

The cost of food F₁ = Rs 4 per unit

Food F₂ = Rs 6 per unit

Thus,

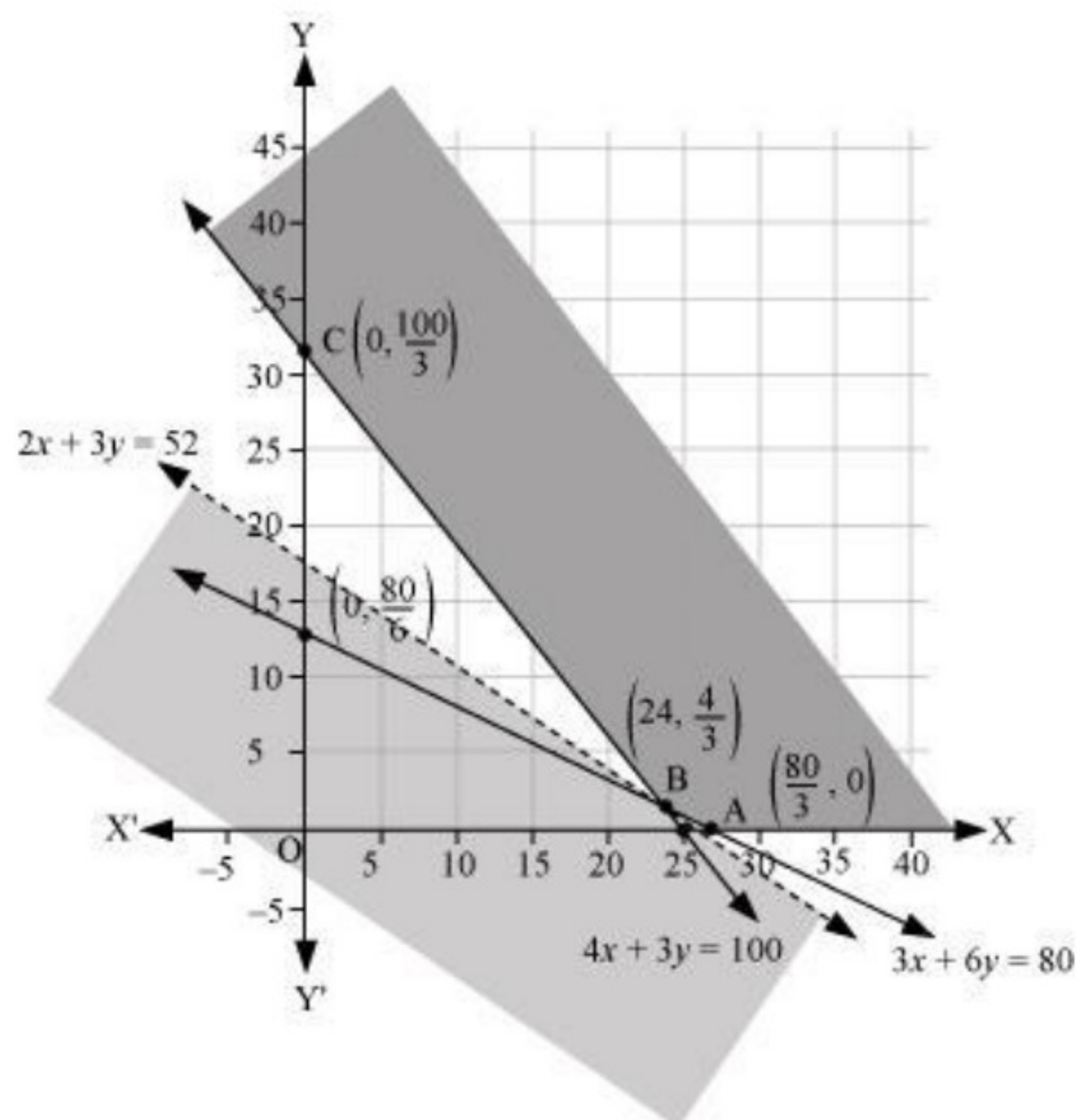
$$3x + 6y \geq 80$$

$$4x + 3y \geq 100$$

$$x, y \geq 0$$

$$\text{Total cost of the diet, } Z = 4x + 6y$$

Now,



It can be seen that the feasible region is unbounded.

The corner points are:

$$A = \left(\frac{80}{3}, 0 \right)$$

$$B = \left(24, \frac{4}{3} \right)$$

$$C = \left(0, \frac{100}{3} \right)$$

Corner point	$Z = 4x + 6y$
$A\left(\frac{80}{3}, 0\right)$	$\frac{320}{3} = 106.6$

$B\left(24, \frac{4}{3}\right)$	104
$C\left(0, \frac{100}{3}\right)$	200

The values of Z are:

We can see that the feasible region has no common point with $2x + 3y < 52$

Therefore, the minimum cost of the mixture will be Rs 104.

- 10.** There are two types of fertilizers F_1 and F_2 . F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 cost Rs 6/kg and F_2 costs Rs 5/kg, determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

Sol.

Let the farmer buy fertilizer $F_1 = x$ kg

Fertilizer $F_2 = y$ kg

As per the question,

	Nitrogen (%)	Phosphoric Acid (%)	Cost (Rs/kg)
$F_1 (x)$	10	6	6
$F_2 (y)$	5	10	5
Requirement (kg)	14	14	

F_1 consists of 10% nitrogen

F₂ consists of 5% nitrogen

But, the farmer requires at least 14 kg of nitrogen.

$$\frac{x}{10} + \frac{y}{20} \geq 14$$

$$2x + y \geq 280$$

F₁ consists of 6% phosphoric acid

F₂ consists of 10% phosphoric acid

But, the farmer requires at least 14 kg of phosphoric acid.

$$\frac{6x}{100} + \frac{10y}{100} \geq 14$$

$$3x + 5y \geq 700$$

Total cost of fertilizers, $Z = 6x + 5y$

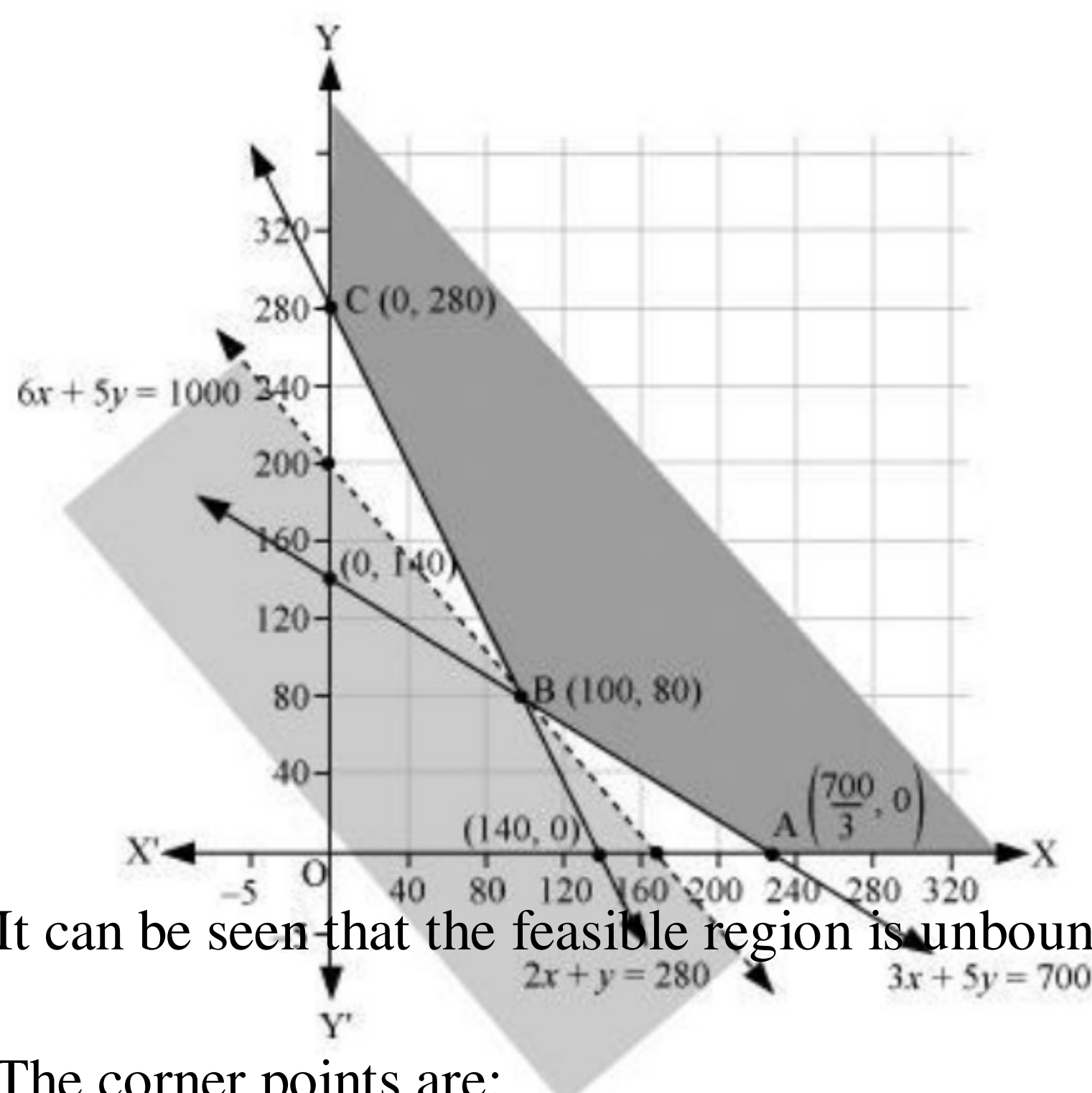
The equations are:

$$2x + y \geq 280$$

$$3x + 5y \geq 700$$

$$x, y \geq 0$$

Now,



It can be seen that the feasible region is unbounded.

The corner points are:

$$A = \left(\frac{700}{3}, 0 \right)$$

$$B = (100, 80)$$

$$C = (0, 280)$$

The values of Z are:

Corner point	$Z = 6x + 5y$
$A\left(\frac{700}{3}, 0\right)$	1400
B(100, 80)	1000
C(0, 280)	1400

It can be seen that the feasible region has no common point with

$$6x + 5y < 1000$$

Therefore, 100 kg of fertiliser F_1 and 80 kg of fertilizer F_2 should be used to minimize the cost.

The minimum cost = Rs 1000

11. The corner points of the feasible region determined by the following system of linear inequalities: $2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$ are $(0, 0)$, $(5, 0)$, $(3, 4)$ and $(0, 5)$. Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both $(3, 4)$ and $(0, 5)$ is

(A) $p = q$

(B) $p = 2q$

(C) $p = 3q$

(D) $q = 3p$

Sol.

Maximum value of Z occurs at two points, $(3, 4)$ and $(0, 5)$.

\therefore Value of Z at $(3, 4)$ = Value of Z at $(0, 5)$

$$p(3) + q(4) = p(0) + q(5)$$

$$3p + 4q = 5q$$

$$\Rightarrow q = 3p$$

Hence, the correct answer is D.