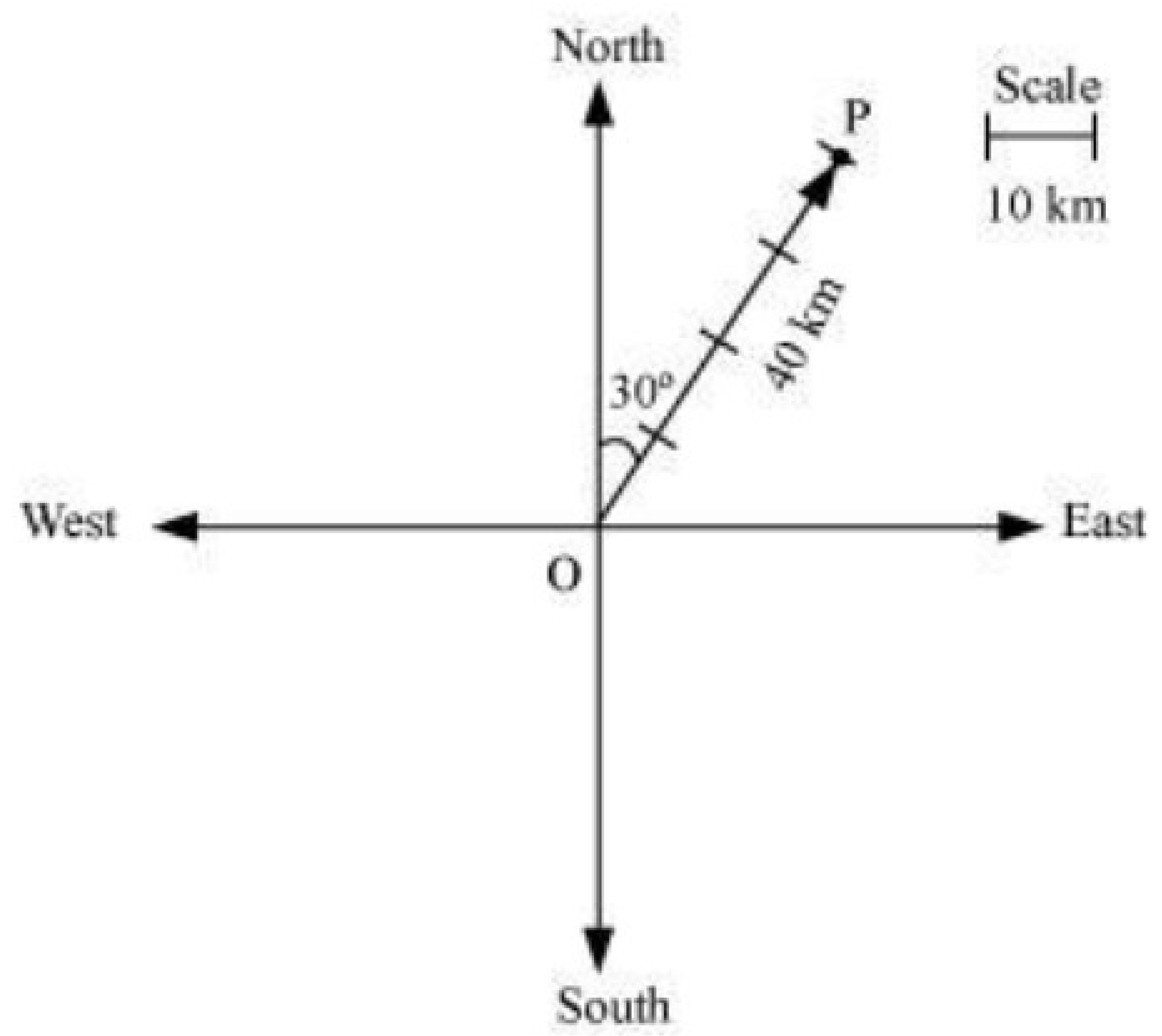
Chapter - 10
Vector Algebra
Class – XII
Subject –Maths

Exersise-10.1

1. Represent graphically a displacement of 40 km, 30° east of north.

Sol.



Thus, $\overrightarrow{Op}$ represents the displacement of 40 km, 30° East of North.

2. Classify the following measures as scalars and vectors.

(i) 10 kg

(ii) 2 metres north-west

(iii) 40°

(iv) 40 watt

(v) 10^{-19} **coulomb**

(vi) **20** m/s^2

Sol.

(i) It is a scalar quantity as it has only magnitude.

(ii) It is a vector quantity as it has both magnitude and direction.

(iii) It is a scalar quantity as it has only magnitude.

(iv) It is a scalar quantity as it has only magnitude.

(v) It is a scalar quantity as it has only magnitude.

(vi) It is a vector quantity as it has both magnitude as well as direction.

3. Classify the following as scalar and vector quantities.

(i) Time period

(ii) Distance

(iii) Force

(iv) Velocity

(v) **Work done**

Sol.

(i) It is a scalar quantity as it has only magnitude.

(ii) It is a scalar quantity as it has only magnitude.

(iii) It is a vector quantity as it has both magnitude and direction.

(iv) It is a vector quantity as it has both magnitude as well as direction.

(v) It is a scalar quantity as it has only magnitude.

4. In Figure, identify the following vectors.

(i) Coinitial

(ii) Equal

(iii) Collinear but not equal

Sol.

(i) Vectors a & d are coinitial because they have the same initial point.

(ii) Vectors b & d are equal because they have the same magnitude and direction.

(iii) Vectors a & C are collinear but not equal because they are parallel, but their directions are not the same.

5. Answer the following as true or false.

(i) $\vec{a}$ & - $\vec{a}$ are collinear

(ii) Two collinear vectors are always equal in magnitude.

(iii) Two vectors having same magnitude are collinear.

(iv) Two collinear vectors having the same magnitude are equal.

Sol.

(i) True because vectors $\vec{a}$ & -$\vec{a}$ are parallel to the same line.

(ii) False because collinear vectors are those vectors that are parallel to the same line.

(iii) False because it is not necessary that vectors having the same magnitude are parallel to each other.

(iv) False because two vectors are equal if they have the same magnitude and direction.

Exersise-10.2

1. Compute the magnitude of the following vectors:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$
$$\vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}$$
$$\vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

Sol.

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$
$$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$\vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}$$
$$|\vec{b}| = \sqrt{2^2 + (-7)^2 + (-3)^2} = \sqrt{62}$$

$$\vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$
$$|\vec{c}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = 1$$

2. Write two different vectors having same magnitude.

Sol.

$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$$

$$|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9+4+1} = \sqrt{14}$$

The vectors are different because they have different directions.

3. Write two different vectors having same direction.

Sol.

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$

$$l = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}$$

$$m = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}$$

$$n = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}$$

$$\vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$l = \frac{3}{\sqrt{3^2+3^2+3^2}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$m = \frac{3}{\sqrt{3^2+3^2+3^2}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$n = \frac{3}{\sqrt{3^2+3^2+3^2}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Hence, the two vectors have the same direction.

4. Find the values of *x* and *y* so that the vectors $2\hat{i}+3\hat{j}$ and $x\hat{i}+y\hat{j}$ are equal

Sol.

$$2\hat{i}+3\hat{j}$$
$$x\hat{i}+y\hat{j}$$

The two vectors will be equal if their corresponding components are equal. Thus,

$x=2, y=3$

5. Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (–5, 7).

Sol.

Initial point P = (2, 1)

Terminal point Q = (–5, 7)

$$\overrightarrow{PQ}=(-5-2)\hat{i}+(7-1)\hat{j}$$
$$\overrightarrow{PQ}=-7\hat{i}+6\hat{j}$$

Hence, the scalar components are = –7 and 6

And the vector components are $-7\hat{i}$ & $6\hat{j}$

6. Find the sum of the vectors:

$$\vec{a}=\hat{i}-2\hat{j}+\hat{k}$$
$$\vec{b}=-2\hat{i}+4\hat{j}+5\hat{k}$$
$$\vec{c}=\hat{i}-6\hat{j}-7\hat{k}$$

Sol.

$$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$$
$$\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$$
$$\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$$

$$\vec{a} + \vec{b} + \vec{c} = (1 - 2 + 1)\hat{i} + (-2 + 4 - 6)\hat{j} + (1 + 5 - 7)\hat{k}$$
$$\vec{a} + \vec{b} + \vec{c} = 0\hat{i} - 4\hat{j} - \hat{k}$$
$$\vec{a} + \vec{b} + \vec{c} = -4\hat{j} - \hat{k}$$

7. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

Sol.

$$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$$
$$\left|\vec{a}\right| = \sqrt{1^2 + 1^2 + 2^2}$$
$$= \sqrt{6}$$

unit vector, $\hat{a} = \dfrac{\vec{a}}{\left|\vec{a}\right|}$

$$= \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}}$$

$$= \frac{\hat{i}}{\sqrt{6}} + \frac{\hat{j}}{\sqrt{6}} + \frac{2\hat{k}}{\sqrt{6}}$$
$$= \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

8. Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points (1, 2, 3) and (4, 5, 6), respectively.

Sol.

P = (1, 2, 3)

Q = (4, 5, 6)

$$\overrightarrow{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k}$$
$$\overrightarrow{PQ} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$\left|\overrightarrow{PQ}\right| = \sqrt{3^2 + 3^2 + 3^2}$$
$$= 3\sqrt{3}$$

unit vector, $\widehat{PQ} = \dfrac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|}$

$$= \frac{3\hat{i} + 3\hat{j} + 3\hat{k}}{3\sqrt{3}}$$
$$= \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

9. For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$

Sol.

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$$
$$\vec{b} = -\hat{i} + \hat{j} - \hat{k}$$

$$\vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k}$$
$$\vec{a} + \vec{b} = \hat{i} + 0\hat{j} + \hat{k}$$
$$\vec{a} + \vec{b} = \hat{i} + \hat{k}$$

$$\left|\vec{a} + \vec{b}\right| = \sqrt{1^2 + 1^2}$$
$$= \sqrt{2}$$

$$\text{unit vector, } \left(\vec{a} + \vec{b}\right) = \frac{\left(\vec{a} + \vec{b}\right)}{\left|\vec{a} + \vec{b}\right|}$$
$$= \frac{\hat{i} + \hat{k}}{\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$$

10. Find a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.

Sol.

$$\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$$
$$\left|\vec{a}\right| = \sqrt{5^2 + (-1)^2 + 2^2}$$
$$= \sqrt{30}$$
$$\text{unit vector, } \hat{a} = \frac{\left(\vec{a}\right)}{\left|\vec{a}\right|}$$
$$= \frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}$$

Thus, the vector in the direction of given vector is,

$$8\vec{a} = 8\left(\frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}\right)$$

$$= \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$$

11. Show that the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ & $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

Sol.

$$\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$$

$$\vec{b} = -2\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right)$$

$$\vec{b} = -2\vec{a}$$

Hence, the given vectors are collinear.

12. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$

Sol.

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2}$$

$$= \sqrt{14}$$

Thus, direction cosines of $\vec{a}$ are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

13. Find the direction cosines of the vector joining the points A (1, 2, –3) and B (–1, –2, 1) directed from A to B.

Sol.

A = (1, 2, –3)

B = (–1, –2, 1)

$$\overrightarrow{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + (1+3)\hat{k}$$
$$\overrightarrow{AB} = -2\hat{i} - 4\hat{j} + 4\hat{k}$$
$$\left|\overrightarrow{AB}\right| = \sqrt{(-2)^2 + (-4)^2 + 4^2}$$
$$= \sqrt{4+16+16}$$
$$= \sqrt{6}$$

Thus, direction cosines of $\overrightarrow{AB}$ are $\left(-\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}\right) = \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$

14. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY, and OZ.

Sol.

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$
$$\left|\vec{a}\right| = \sqrt{1^2 + 1^2 + 1^2}$$
$$= \sqrt{3}$$

Thus, direction cosines of $\vec{a}$ are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Now,

Let α, β, and γ = the angles formed by $\vec{a}$ with the positive directions of x, y, and z axes.

$$\cos\alpha = \frac{1}{\sqrt{3}}$$

$$\cos\beta = \frac{1}{\sqrt{3}}$$

$$\cos\gamma = \frac{1}{\sqrt{3}}$$

Thus, $\vec{a}$ is equally inclined to axes OX, OY, and OZ.

15. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ & $-\hat{i}+\hat{j}+\hat{k}$ respectively, in the ration 2:1

(i) Internally

(ii) Externally

Sol.

$$\overrightarrow{OP} = \hat{i}+2\hat{j}-\hat{k}$$

$$\overrightarrow{OQ} = -\hat{i}+\hat{j}+\hat{k}$$

(i) The position vector of point R which divides the line joining two points P and Q internally in the ratio 2:1 is,

$$= \frac{m\vec{b}+n\vec{a}}{m+n}$$

$$\overrightarrow{OR}=\frac{2\left(\hat{i}+2\hat{j}-\hat{k}\right)+1\left(-\hat{i}+\hat{j}+\hat{k}\right)}{2+1}$$
$$=\frac{\left(2\hat{i}+4\hat{j}-2\hat{k}\right)+\left(-\hat{i}+\hat{j}+\hat{k}\right)}{3}$$
$$=\frac{-\hat{i}+4\hat{j}+\hat{k}}{3}$$
$$=\frac{-1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$$

(ii) The position vector of point R which divides the line joining two points P and Q externally in the ratio 2:1 is given by,

$$=\frac{m\vec{b}-n\vec{a}}{m-n}$$

$$\overrightarrow{OR}=\frac{2\left(-\hat{i}+\hat{j}+\hat{k}\right)-1\left(\hat{i}+2\hat{j}-\hat{k}\right)}{2-1}$$
$$=\frac{\left(-2\hat{i}+2\hat{j}+2\hat{k}\right)-\left(\hat{i}+2\hat{j}-\hat{k}\right)}{1}$$
$$=-3\hat{i}+3\hat{k}$$

16. Find the position vector of the midpoint of the vector joining the points P (2, 3, 4) and Q (4, 1, – 2).

Sol.

P = (2, 3, 4)

Q = (4, 1, – 2)

$$\overrightarrow{OR} = \frac{\left(2\hat{i}+3\hat{j}+4\hat{k}\right)+\left(4\hat{i}+\hat{j}-2\hat{k}\right)}{2}$$
$$= \frac{(2+4)\hat{i}+(3+1)\hat{j}+(4-2)\hat{k}}{2}$$
$$= \frac{6\hat{i}+4\hat{j}+2\hat{k}}{2}$$
$$= 3\hat{i}+2\hat{j}+\hat{k}$$

17. Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k},$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ respectively form the vertices of a right angled triangle.

Sol.

$$\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$$
$$\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$$
$$\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$\overrightarrow{AB} = \vec{b} - \vec{a}$$
$$\overrightarrow{AB} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k}$$
$$\overrightarrow{AB} = -\hat{i} + 3\hat{j} + 5\hat{k}$$
$$\left|\overrightarrow{AB}\right| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{35}$$

$$\overrightarrow{BC} = \vec{c} - \vec{b}$$
$$\overrightarrow{BC} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k}$$
$$\overrightarrow{BC} = -\hat{i} - 2\hat{j} - 6\hat{k}$$
$$\left|\overrightarrow{BC}\right| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{41}$$

$\overrightarrow{CA} = \vec{a} - \vec{c}$

$\overrightarrow{CA} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k}$

$\overrightarrow{CA} = 2\hat{i} - \hat{j} + \hat{k}$

$\left|\overrightarrow{CA}\right| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$

$\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{CA}\right|^2 = \left|\overrightarrow{BC}\right|^2$

$35 + 6 = 41$

Hence, ABC is a right-angled triangle.

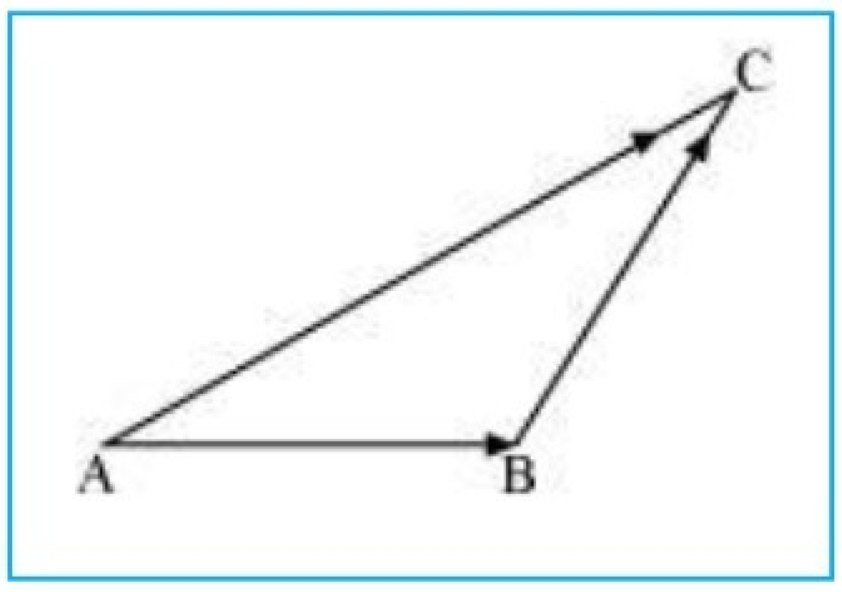
18. In triangle ABC which of the following is not true:

A. $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} + \overrightarrow{\mathbf{CA}} = \vec{\mathbf{0}}$

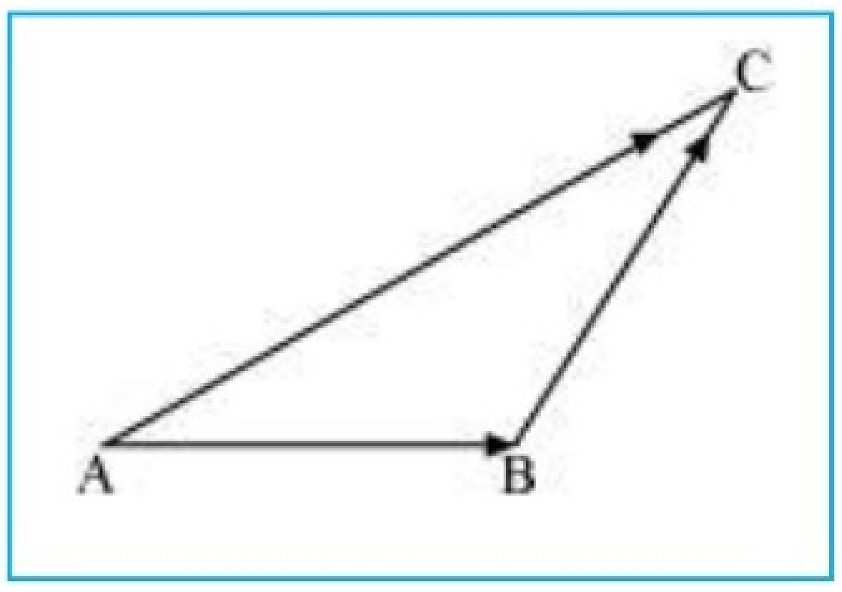
B. $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} - \overrightarrow{\mathbf{AC}} = \vec{\mathbf{0}}$

C. $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} - \overrightarrow{\mathbf{CA}} = \vec{\mathbf{0}}$

D. $\overrightarrow{\mathbf{AB}} - \overrightarrow{\mathbf{CB}} + \overrightarrow{\mathbf{CA}} = \vec{\mathbf{0}}$



Sol.



On applying the triangle law of addition in the given triangle,

$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

$\overrightarrow{AB}+\overrightarrow{BC}=-\overrightarrow{CA}$

$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=0$

Thus, A is true

Now,

$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

$\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=0$

Thus, B is true

Now,

$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=0$

$\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=0$

Thus, D is true

Hence, C is false

The correct answer is C.

19. If $\vec{a}$ & $\vec{b}$ are two collinear vectors, then which of the following are incorrect:

A. $\vec{b}=\pm\lambda$, for some scalar λ

B. $\vec{a}=\pm\vec{b}$

C. the respective components of $\vec{a}$ & $\vec{b}$ are proportional

D. both the vectors $\vec{a}$ & $\vec{b}$ have same direction, but different magnitudes

Sol.

If $\vec{a}$ & $\vec{b}$ are two collinear vectors, then they are parallel.

$$\vec{b} = \lambda\vec{a}$$
$$\lambda = \pm 1$$
$$\vec{b} = \pm\vec{a}$$

$$\vec{a} == a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$
$$\vec{b} == b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{b} = \lambda\vec{a}$$
$$b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda\left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$$
$$b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda a_1\hat{i} + \lambda a_2\hat{j} + \lambda a_3\hat{k}$$

$$b_1 = \lambda a_1$$
$$b_2 = \lambda a_2$$
$$b_3 = \lambda a_3$$

$$\frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3}$$

Thus, the respective components of $\vec{a}$ & $\vec{b}$ are proportional. These vectors can have different directions.

Hence, the statement given in D is incorrect.

The correct answer is D.

Exercise-10.3

1. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2, respectively having $\vec{a}.\vec{b} = \sqrt{6}$.

Sol.

$$\left|\vec{a}\right| = \sqrt{3}$$
$$\left|\vec{b}\right| = 2$$
$$\vec{a}.\vec{b} = \sqrt{6}$$

$$\vec{a}.\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\cos\theta$$
$$\sqrt{6} = \sqrt{3}.2.\cos\theta$$
$$\cos\theta = \frac{\sqrt{6}}{2\sqrt{3}}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$
$$\theta = \frac{\pi}{4}$$

2. Find the angle between the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

Sol.

$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$
$$\left|\vec{a}\right| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}$$

$$\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$$
$$\left|\vec{b}\right| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{14}$$

$$\vec{a}.\vec{b}=\left(\hat{i}-2\hat{j}+3\hat{k}\right)\left(3\hat{i}-2\hat{j}+\hat{k}\right)$$
$$=1.3+(-2).(-2)+3.1$$
$$=10$$

Also,

$$\vec{a}.\vec{b}=\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta$$
$$10=\sqrt{14}.\sqrt{14}.\cos\theta$$
$$\cos\theta=\frac{10}{14}$$
$$\theta=\cos^{-1}\left(\frac{5}{7}\right)$$

3. Find the projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$.

Sol.

$$\vec{a}=\hat{i}-\hat{j}$$
$$\vec{b}=\hat{i}+\hat{j}$$

$$\text{Projection of } \vec{a} \text{ on } \vec{b}=\frac{1}{\left|\vec{b}\right|}\left(\vec{a}.\vec{b}\right)$$
$$=\frac{1}{\sqrt{1+1}}\left[1.1+(-1).1\right]$$
$$=0$$

Thus, Projection of $\vec{a}$ *on* $\vec{b}=0$

4. Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$.

Sol.

$$\vec{a}=\hat{i}+3\hat{j}+7\hat{k}$$
$$\vec{b}=7\hat{i}-\hat{j}+8\hat{k}$$
$$|\vec{b}|=\sqrt{7^2+(-1)^2+8^2}$$
$$=\sqrt{49+1+64}$$
$$=\sqrt{114}$$

$$\vec{a}.\vec{b}=[1.7+3.(-1)+7.8]$$
$$=60$$

$$\text{Projection of } \vec{a} \text{ on } \vec{b}=\frac{1}{|\vec{b}|}(\vec{a}.\vec{b})$$
$$=\frac{60}{\sqrt{114}}$$

5. Show that each of the given three vectors is a unit vector:

$$\frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k}),\frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k}),\text{and }\frac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k})$$

Also, show that they are mutually perpendicular to each other.

Sol.

$$\vec{a}=\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right)=\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}$$

$$\left|\vec{a}\right|=\sqrt{\left(\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2+\left(\frac{6}{7}\right)^2}=\sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}}=\sqrt{\frac{49}{49}}=1$$

$$\vec{b}=\frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right)=\frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}$$

$$\left|\vec{b}\right|=\sqrt{\left(\frac{3}{7}\right)^2+\left(\frac{-6}{7}\right)^2+\left(\frac{2}{7}\right)^2}=\sqrt{\frac{9}{49}+\frac{36}{49}+\frac{4}{49}}=\sqrt{\frac{49}{49}}=1$$

$$\vec{c}=\frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)=\frac{6}{7}\hat{i}+\frac{2}{7}\hat{j}-\frac{3}{7}\hat{k}$$

$$\left|\vec{c}\right|=\sqrt{\left(\frac{6}{7}\right)^2+\left(\frac{2}{7}\right)^2+\left(\frac{-3}{7}\right)^2}=\sqrt{\frac{36}{49}+\frac{4}{49}+\frac{9}{49}}=\sqrt{\frac{49}{49}}=1$$

Thus, the given three vectors are a unit vector.

Now,

$$\vec{a}.\vec{b}=\frac{2}{7}\times\frac{3}{7}+\frac{3}{7}\times\frac{-6}{7}+\frac{6}{7}\times\frac{2}{7}=\frac{6}{49}-\frac{18}{49}+\frac{12}{49}=0$$

$$\vec{b}.\vec{c}=\frac{3}{7}\times\frac{6}{7}+\frac{-6}{7}\times\frac{2}{7}+\frac{2}{7}\times\frac{-3}{7}=\frac{18}{49}-\frac{12}{49}-\frac{6}{49}=0$$

$$\vec{c}.\vec{a}=\frac{6}{7}\times\frac{2}{7}+\frac{2}{7}\times\frac{3}{7}+\frac{-3}{7}\times\frac{6}{7}=\frac{12}{49}+\frac{6}{49}-\frac{18}{49}=0$$

Hence, the given three vectors are mutually perpendicular to each other.

6. **Find $|\vec{a}|$ and $|\vec{b}|$, if $|\vec{a}| = 8|\vec{b}|$ and $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 8$.**

Sol.

$$|\vec{a}| = 8|\vec{b}|$$

$$(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 8$$
$$\vec{a}.\vec{a} - \vec{a}.\vec{b} + \vec{b}.\vec{a} - \vec{b}.\vec{b} = 8$$
$$|\vec{a}|^2 - |\vec{b}|^2 = 8$$
$$\left[8|\vec{b}|\right]^2 - |\vec{b}|^2 = 8$$

$$63|\vec{b}|^2 = 8$$
$$|\vec{b}|^2 = \frac{8}{63}$$
$$|\vec{b}| = \frac{2\sqrt{2}}{3\sqrt{7}}$$

$$|\vec{a}| = 8|\vec{b}|$$
$$|\vec{a}| = 8\left[\frac{2\sqrt{2}}{3\sqrt{7}}\right]$$
$$|\vec{a}| = \frac{16\sqrt{2}}{3\sqrt{7}}$$

7. **Evaluate the product $(3\vec{a} - 5\vec{b}).(2\vec{a} + 7\vec{b})$.**

Sol.

$$\left(3\vec{a}-5\vec{b}\right).\left(2\vec{a}+7\vec{b}\right)$$
$$=3\vec{a}.2\vec{a}+3\vec{a}.7\vec{b}-5\vec{b}.2\vec{a}-5\vec{b}.7\vec{b}$$
$$=6\left|\vec{a}\right|^2+21\vec{a}.\vec{b}-10\vec{a}.\vec{b}-35\left|\vec{b}\right|^2$$
$$=6\left|\vec{a}\right|^2+11\vec{a}.\vec{b}-35\left|\vec{b}\right|^2$$

8. Find the magnitude of two vectors $\left|\vec{a}\right|$ and $\left|\vec{b}\right|$, having the same magnitude and such that the angle between them is 60° and their scalar product is $\frac{1}{2}$.

Sol.

Let θ = angle between the vectors

$$\left|\vec{a}\right|=\left|\vec{b}\right|$$
$$\vec{a}.\vec{b}=\frac{1}{2}$$
$$\theta=60^\circ$$

$$\vec{a}.\vec{b}=\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta$$
$$\frac{1}{2}=\left|\vec{a}\right|.\left|\vec{a}\right|\cos 60^\circ$$
$$\frac{1}{2}=\left|\vec{a}\right|^2.\frac{1}{2}$$
$$\left|\vec{a}\right|^2=1$$
$$\left|\vec{a}\right|=1$$
$$\left|\vec{a}\right|^2=\left|\vec{b}\right|=1$$

9. Find $\left|\vec{x}\right|$, if for a unit vector $\left(\vec{x}-\vec{a}\right).\left(\vec{x}+\vec{a}\right)=12$.

Sol.

$$\left(\vec{x}-\vec{a}\right).\left(\vec{x}+\vec{a}\right)=12$$
$$\vec{x}.\vec{x}+\vec{x}.\vec{a}-\vec{a}.\vec{x}-\vec{a}.\vec{a}=12$$
$$\left|\vec{x}\right|^2-\left|\vec{a}\right|^2=12$$
$$\left|\vec{x}\right|^2-1=12$$
$$\left|\vec{x}\right|^2=13$$
$$\left|\vec{x}\right|=\sqrt{13}$$

10. If $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

Sol.

$$\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}$$
$$\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$$
$$\vec{c}=3\hat{i}+\hat{j}$$

$$\vec{a}+\lambda\vec{b}=\left(2\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(-\hat{i}+2\hat{j}+\hat{k}\right)$$
$$=(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}$$

Now,

$\left(\vec{a}+\lambda\vec{b}\right)$ is perpendicular to $\vec{c}$

$$\therefore\left(\vec{a}+\lambda\vec{b}\right).\vec{c}=0$$
$$\left[(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}\right]\left(3\hat{i}+\hat{j}\right)=0$$
$$(2-\lambda)3+(2+2\lambda)1+(3+\lambda)0=0$$
$$6-3\lambda+2+2\lambda=0$$
$$8-\lambda=0$$
$$\lambda=8$$

Hence, the required value of λ is 8.

11. Show that $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$, for any two nonzero vectors $\vec{a}$ and $\vec{b}$

Sol.

$$\left(|\vec{a}|\vec{b}+|\vec{b}|\vec{a}\right).\left(|\vec{a}|\vec{b}-|\vec{b}|\vec{a}\right)$$
$$=|\vec{a}|^2\,\vec{b}.\vec{b}-|\vec{a}|\,|\vec{b}|\vec{b}.\vec{a}+|\vec{b}|\,|\vec{a}|\,\vec{a}.\vec{b}-|\vec{b}|^2\,\vec{a}.\vec{a}$$
$$=|\vec{a}|^2\,|\vec{b}|^2-|\vec{b}|^2\,|\vec{a}|^2$$
$$=0$$

Hence, $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ and $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$ are perpendicular to each other.

12. If $\vec{a}.\vec{a}=0$ and $\vec{a}.\vec{b}=0$, then what can be concluded about the vector $\vec{b}$?

Sol.

$$\vec{a}.\vec{a}=0$$
$$\vec{a}.\vec{b}=0$$

Now,

$$\vec{a}.\vec{a}=0$$
$$|\vec{a}|^2=0$$
$$\vec{a}=0$$

Thus, $\vec{a}$ is a zero vector & $\vec{b}$ can be any vector.

13. **If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$**

Sol.

$$\vec{a}+\vec{b}+\vec{c}=\vec{0}$$
$$\vec{a}.\left(\vec{a}+\vec{b}+\vec{c}\right)=\vec{a}.\vec{0}$$
$$\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}=\vec{a}.0$$
$$1+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0.................................(1)$$

$$\vec{a}+\vec{b}+\vec{c}=\vec{0}$$
$$\vec{b}.\left(\vec{a}+\vec{b}+\vec{c}\right)=\vec{b}.\vec{0}$$
$$\vec{b}.\vec{a}+\vec{b}.\vec{b}+\vec{b}.\vec{c}=\vec{b}.0$$
$$\vec{b}.\vec{a}+1+\vec{b}.\vec{c}=0.................................(2)$$

$$\vec{a}+\vec{b}+\vec{c}=\vec{0}$$
$$\vec{c}.\left(\vec{a}+\vec{b}+\vec{c}\right)=\vec{c}.\vec{0}$$
$$\vec{c}.\vec{a}+\vec{c}.\vec{b}+\vec{c}.\vec{c}=\vec{c}.0$$
$$\vec{c}.\vec{a}+\vec{c}.\vec{b}+1=0.................................(3)$$

Now, add all three equations

$$1+\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{a}+1+\vec{b}.\vec{c}+\vec{c}.\vec{a}+\vec{c}.\vec{b}+1=0$$
$$3+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$$
$$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-\frac{3}{2}$$

14. **If either vector $\vec{a}=0$ or $\vec{b}=0$, then $\vec{a}.\vec{b}=0$, But the converse need not be true. Justify your answer with an example.**

Sol.

$$\vec{a} = 2\hat{i} + 4\hat{j} + 3\hat{k}$$
$$|\vec{a}| = \sqrt{2^2 + 4^2 + 3^2} = \sqrt{29}$$
$$\vec{b} = 3\hat{i} + 3\hat{j} - 6\hat{k}$$
$$|\vec{b}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{54}$$
$$\vec{a}.\vec{b} = 2.3 + 4.3 + 3(-6) = 6 + 12 - 18 = 0$$

Thus, $\vec{a} \neq 0$ & $\vec{b} \neq 0$

Hence, the converse of the given statement need not be true.

15. If the vertices A, B, C of a triangle ABC are (1, 2, 3), (–1, 0, 0), (0, 1, 2), respectively, then find ∠ABC. [∠ABC is the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$]

Sol.

The vertices of ΔABC are:

A = (1, 2, 3)

B = (–1, 0, 0)

C = (0, 1, 2)

$$\overrightarrow{BA} = [1 - (-1)]\hat{i} + (2 - 0)\hat{j} + (3 - 0)\hat{k}$$
$$= 2\hat{i} + 2\hat{j} + 3\hat{k}$$
$$|\overrightarrow{BA}| = \sqrt{2^2 + 2^2 + 3^2} = \sqrt{17}$$

$$\overrightarrow{BC} = [0-(-1)]\hat{i} + (1-0)\hat{j} + (2-0)\hat{k}$$
$$= \hat{i} + \hat{j} + 2\hat{k}$$
$$|\overrightarrow{BC}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

$$\overrightarrow{BA}.\overrightarrow{BC} = (2\hat{i} + 2\hat{j} + 3\hat{k})(\hat{i} + \hat{j} + 2\hat{k})$$
$$= 2.1 + 2.1 + 3.2$$
$$= 10$$

Now,

$$\overrightarrow{BA}.\overrightarrow{BC} = |\overrightarrow{BA}|.|\overrightarrow{BC}|\cos(\angle ABC)$$
$$10 = \sqrt{17} \times \sqrt{6}\cos(\angle ABC)$$
$$\cos(\angle ABC) = \frac{10}{\sqrt{102}}$$
$$\angle ABC = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$$

16. Show that the points A (1, 2, 7), B (2, 6, 3) and C (3, 10, –1) are collinear.

Sol.

A = (1, 2, 7)

B = (2, 6, 3)

C = (3, 10, –1)

$$\overrightarrow{AB} = (2-1)\hat{i} + (6-2)\hat{j} + (3-7)\hat{k}$$
$$= \hat{i} + 4\hat{j} - 4\hat{k}$$
$$|\overrightarrow{AB}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{33}$$

$$\overrightarrow{BC} = (3-2)\hat{i} + (10-6)\hat{j} + (-1-3)\hat{k}$$
$$= \hat{i} + 4\hat{j} - 4\hat{k}$$
$$\left|\overrightarrow{BC}\right| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{33}$$

$$\overrightarrow{AC} = (3-1)\hat{i} + (10-2)\hat{j} + (-1-7)\hat{k}$$
$$= 2\hat{i} + 8\hat{j} - 8\hat{k}$$
$$\left|\overrightarrow{AC}\right| = \sqrt{2^2 + 8^2 + (-8)^2} = 2\sqrt{33}$$

Thus,
$$\left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$$

Hence, the given points A, B, and C are collinear.

17. Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right angled triangle.

Sol.

Let $\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$
$$\vec{B} = \hat{i} - 3\hat{j} - 5\hat{k}$$
$$\vec{C} = 3\hat{i} - 4\hat{j} - 4\hat{k}$$

$$\overrightarrow{AB} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k}$$
$$= -\hat{i} - 2\hat{j} - 6\hat{k}$$
$$\left|\overrightarrow{AB}\right| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{41}$$

$$\overrightarrow{BC} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k}$$
$$= 2\hat{i} - \hat{j} + \hat{k}$$
$$\left|\overrightarrow{BC}\right| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$$

$$\overrightarrow{AC} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k}$$
$$= -\hat{i} + 3\hat{j} + 5\hat{k}$$
$$\left|\overrightarrow{AC}\right| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{35}$$

$$\therefore \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{AC}\right|^2 = \left|\overrightarrow{AB}\right|$$
$$6 + 35 = 41$$

Hence, ΔABC is a right-angled triangle.

18. If $\vec{a}$ is a nonzero vector of magnitude 'a' and λ a nonzero scalar, then $\lambda\vec{a}$ is unit vector if

(A) λ = 1

(B) λ = –1

(C) a = |λ|

(D) a = $\frac{1}{|\lambda|}$

Sol.

$$|\lambda\vec{a}|=1$$
$$|\lambda||\vec{a}|=1$$
$$|\vec{a}|=\frac{1}{|\lambda|}$$
$$a=\frac{1}{|\lambda|}$$

The correct answer is D.

Exercise-10.4

1. Find $|\vec{a}\times\vec{b}|$, if $\vec{a}=\hat{i}-7\hat{j}+7\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$.

Sol.

$$\vec{a}=\hat{i}-7\hat{j}+7\hat{k}$$
$$\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -7 & 7\\ 3 & -2 & 2\end{vmatrix}$$
$$=(-14+14)\hat{i}-(2-21)\hat{j}+(-2+21)\hat{k}$$
$$=19\hat{j}+19\hat{k}$$

$$|\vec{a}\times\vec{b}|=\sqrt{19^2+19^2}=19\sqrt{2}$$

2. Find a unit vector perpendicular to each of the vector $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$.

Sol.

$$\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$$
$$\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$$

$$\vec{a}+\vec{b}=4\hat{i}+4\hat{j}$$
$$\vec{a}-\vec{b}=2\hat{i}+4\hat{k}$$

$$\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 4 & 0\\ 2 & 0 & 4\end{vmatrix}$$
$$=16\hat{i}-16\hat{j}-8\hat{k}$$

$$\left|\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right|=\sqrt{16^2+\left(-16\right)^2+\left(-8\right)}$$
$$=24$$

Thus, the unit vector perpendicular to the vectors is,

$$=\pm\frac{\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)}{\left|\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right|}$$
$$=\pm\frac{16\hat{i}-16\hat{j}-8\hat{k}}{24}$$
$$=\pm\frac{2\hat{i}-2\hat{j}-\hat{k}}{3}$$

3. If a unit vector $\vec{a}$ makes an angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find θ and hence, the compounds of $\vec{a}$

Sol.

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{a}$ is a unit vector.

$\therefore |\vec{a}| = 1$

$\sqrt{a_1^2 + a_2^2 + a_3^2} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$

Now,

$$\cos\frac{\pi}{3} = \frac{a_1}{|\vec{a}|}$$

$$\frac{1}{2} = a_1$$

$$\cos\frac{\pi}{4} = \frac{a_2}{|\vec{a}|}$$

$$\frac{1}{\sqrt{2}} = a_2$$

$$\cos\theta = \frac{a_3}{|\vec{a}|}$$

$$a_3 = \cos\theta$$

From equation 1

$$\sqrt{a_1^2 + a_2^2 + a_3^2} = 1$$

$$\left(\frac{1}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+\cos^2\theta=1$$

$$\frac{3}{4}+\cos^2\theta=1$$

$$\cos^2\theta=\frac{1}{4}$$

$$\cos\theta=\frac{1}{2}$$

$$\theta=\frac{\pi}{3}$$

$$\therefore a_3=\cos\frac{\pi}{3}=\frac{1}{2}$$

Hence, the components of $\vec{a}$ are $\left(\frac{1}{2},\frac{1}{\sqrt{2}},\frac{1}{2}\right)$.

4. Show that

$$\left(\vec{\mathbf{a}}-\vec{\mathbf{b}}\right)\times\left(\vec{\mathbf{a}}+\vec{\mathbf{b}}\right)=2\left(\vec{\mathbf{a}}\times\vec{\mathbf{b}}\right)$$

Sol.

$$\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)$$

$$=\left(\vec{a}-\vec{b}\right)\times\vec{a}+\left(\vec{a}-\vec{b}\right)\times\vec{b}$$

$$=\vec{a}\times\vec{a}-\vec{b}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{b}$$

$$=0+\vec{a}\times\vec{b}+\vec{a}\times\vec{b}-0$$

$$=2\left(\vec{a}\times\vec{b}\right)$$

5. **Find λ and μ if $\left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+\lambda\hat{j}+\mu\hat{k}\right)=0$.**

Sol.

$$\left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+\lambda\hat{j}+\mu\hat{k}\right)=0$$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 6 & 27\\ 1 & \lambda & \mu\end{vmatrix}=0$$

$$(6\mu-27\lambda)\hat{i}-(2\mu-27)\hat{j}+(2\lambda-6)\hat{k}=0\hat{i}-0\hat{j}+0\hat{k}$$

Compare the coefficients

$$6\mu-27\lambda=0$$
$$2\mu-27=0$$
$$2\lambda-6=0$$

On solving above equations,

$$\lambda=3, \mu=\frac{27}{2}$$

6. **Given that $\vec{a}.\vec{b}=0$ & $\vec{a}\times\vec{b}=0$. What can you conclude about the vectors $\vec{a}$ & $\vec{b}$?**

Sol.

$$\vec{a}.\vec{b}=0$$

Then,

(i)

Either $|\vec{a}|=0$ or $|\vec{b}|=0$ or $\vec{a}\perp\vec{b}$

$$\vec{a}\times\vec{b}=0$$

(ii)

Either $\left|\vec{a}\right|=0$ or $\left|\vec{b}\right|=0$ or $\vec{a}\,\square\,\vec{b}$

both the vectors cannot be perpendicular & parallel simultaneously

$\therefore \left|\vec{a}\right|=0, \left|\vec{b}\right|=0$

7. Let the vectors $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}, \vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$. Then show that $\vec{a}\times\left(\vec{b}+\vec{c}\right)=\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}\times\vec{c}\right)$

Sol.

$$\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$$
$$\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$$

$$\left(\vec{b}+\vec{c}\right)=\left(b_1+c_1\right)\hat{i}+\left(b_2+c_2\right)\hat{j}+\left(b_3+c_3\right)\hat{k}$$

$$\vec{a}\times\left(\vec{b}+\vec{c}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1+c_1 & b_2+c_2 & b_3+c_3\end{vmatrix}$$

$$\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$$
$$=\left(a_2b_3+a_2c_3-a_3b_2-a_3c_2\right)\hat{i}+\left(-a_1b_3-a_1c_3+a_3b_1+a_3c_1\right)\hat{j}+\left(a_1b_2+a_1c_2-a_2b_1-a_2c_1\right)\hat{k}..........(1)$$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

$$=\left(a_2b_3-a_3b_2\right)\hat{i}+\left(a_3b_1-a_1b_3\right)\hat{j}+\left(a_1b_2-a_2b_1\right)\hat{k}..........(2)$$

$$\vec{a}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ c_1 & c_2 & c_3\end{vmatrix}$$

$$=\left(a_2c_3-a_3c_2\right)\hat{i}+\left(a_3c_1-a_1c_3\right)\hat{j}+\left(a_1c_2-a_2c_1\right)\hat{k}..........(3)$$

Now, add equation 2 & 3

$$\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}\times\vec{c}\right)=\left(a_2b_3+a_2c_3-a_3b_2-a_3c_2\right)\hat{i}+\left(-a_1b_3-a_1c_3+a_3b_1+a_3c_1\right)\hat{j}+\left(a_1b_2+a_1c_2-a_2b_1-a_2c_1\right)\hat{k}$$

Thus, $LHS = RHS$

$$\vec{a}\times\left(\vec{b}+\vec{c}\right)=\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}\times\vec{c}\right)$$

8. If either $|\vec{a}|=0$ or $|\vec{b}|=0$, then $\vec{a}\times\vec{b}=0$, is the converse true? Justify your answer with an example.

Sol.

Take any parallel non-zero vectors so that $\vec{a}\times\vec{b}=0$.

Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + 4\hat{j} + 6\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 4 & 6 \end{vmatrix}$$

$$= (12-12)\hat{i} - (6-6)\hat{j} + (4-4)\hat{k}$$

$$= 0\hat{i} - 0\hat{j} + 0\hat{k} = 0$$

$$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{2^2 + 4^2 + 6^2} = 2\sqrt{14}$$

$$|\vec{a}| \neq 0 \ \& \ |\vec{b}| \neq 0$$

Hence, the converse of the given statement need not be true.

9. Find the area of the triangle with vertices A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5).

Sol.

A = (1, 1, 2)

B = (2, 3, 5)

C = (1, 5, 5)

Sides of ΔABC are:

$$\overrightarrow{AB} = (2-1)\hat{i} + (3-1)\hat{j} + (5-2)\hat{k}$$
$$= \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overrightarrow{BC} = (1-2)\hat{i} + (5-3)\hat{j} + (5-5)\hat{k}$$
$$= -\hat{i} + 2\hat{j}$$

$$\text{area of } \Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{BC}\right|$$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix}$$
$$= (-6)\hat{i} - (3)\hat{j} + (2+2)\hat{k}$$
$$= -6\hat{i} - 3\hat{j} + 4\hat{k}$$
$$\left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = \sqrt{(-6)^2 + (-3)^2 + 4^2} = \sqrt{61}$$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = \frac{\sqrt{61}}{2} \text{ square units}$$

10. Find the area of the parallelogram whose adjacent sides are determined by the vector $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

Sol.

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$$
$$\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

The area of the parallelogram is:

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 3\\ 2 & -7 & 1\end{vmatrix}$$

$$=(-1+21)\hat{i}-(1-6)\hat{j}+(-7+2)\hat{k}$$

$$=20\hat{i}+5\hat{j}-5\hat{k}$$

$$\left|\vec{a}\times\vec{b}\right|=\sqrt{20^2+5^2+5^2}=15\sqrt{2}$$

Hence, the area of the given parallelogram is $15\sqrt{2}$ square units.

11. Let the vectors $\vec{a}$ & $\vec{b}$ be such that $|\vec{a}|=3$ & $|\vec{b}|=\frac{\sqrt{2}}{3}$, then $\vec{a}\times\vec{b}$ is a unit vector, if the angle between $\vec{a}$ & $\vec{b}$ is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Sol.

$$|\vec{a}|=3$$

$$|\vec{b}|=\frac{\sqrt{2}}{3}$$

We know that,

$\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\hat{n}$

Now,

$\vec{a}\times\vec{b}$ is a unit vector if $|\vec{a}\times\vec{b}|=1$

$|\vec{a}\times\vec{b}|=1$

$||\vec{a}||\vec{b}|\sin\theta\hat{n}|=1$

$3.\frac{\sqrt{2}}{3}\sin\theta=0$

$\sin\theta=\frac{1}{\sqrt{2}}$

$\theta=\frac{\pi}{4}$

The correct answer is B.

12. Area of a rectangle having vertices A, B, C, and D with position vectors $-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k},\hat{i}+\frac{1}{2}\hat{j}+4\hat{k},\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$ and $-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$ respectively is

(A) $\frac{1}{2}$

(B) 1

(C) 2

(D) 4

Sol.

Let $\overrightarrow{OA} = -\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$

$\overrightarrow{OB} = \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$

$\overrightarrow{OC} = \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$

$\overrightarrow{OD} = -\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$

$$\overrightarrow{AB} = (1+1)\hat{i} + \left(\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4-4)\hat{k}$$
$$= 2\hat{i}$$

$$\overrightarrow{BC} = (1-1)\hat{i} + \left(-\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4-4)\hat{k}$$
$$= -\hat{j}$$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = -2\hat{k}$$

$$\left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = \sqrt{(-2)^2} = 2$$

Hence, the area of the given rectangle is 2 square units.

The correct answer is C.