Chapter.2
Inverse Trigonometric Functions
Class–XII
Subject–Mathematics

Exercise-2.1

1. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right) = y$

Sol.

Let $\sin^{-1}\left(-\frac{1}{2}\right) = y$

$$\sin y = -\frac{1}{2}$$

$$\sin y = -\sin\left(\frac{\pi}{6}\right)$$

$$\sin y = \sin\left(-\frac{\pi}{6}\right)$$

$$y = -\frac{\pi}{6}$$

2. Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Sol.

Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$

$$\cos y = \frac{\sqrt{3}}{2}$$

$$\cos y = \cos\left(\frac{\pi}{6}\right)$$

$$y = \frac{\pi}{6}$$

3. Find the principal value of $\operatorname{cosec}^{-1}(2)$

Sol.

Let $y = \operatorname{cosec}^{-1}(2)$

$$\operatorname{cosec} y = 2$$

$$\operatorname{cosec} y = \operatorname{cosec}\left(\frac{\pi}{6}\right)$$

$$y = \frac{\pi}{6}$$

4. Find the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$

Sol.

Let $\tan^{-1}\left(-\sqrt{3}\right) = y$

$$\tan y = -\sqrt{3}$$

$$\tan y = -\tan\left(\frac{\pi}{3}\right)$$

$$\tan y = \tan\left(-\frac{\pi}{3}\right)$$

$$y = -\frac{\pi}{3}$$

5. Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$

Sol.

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$

$$\cos y = -\frac{1}{2}$$

$$\cos y = -\cos\left(\frac{\pi}{3}\right)$$

$$\cos y = \cos\left(\pi - \frac{\pi}{3}\right)$$

$$\cos y = \cos\left(\frac{2\pi}{3}\right)$$

$$y = \frac{2\pi}{3}$$

6. Find the principal value of $\tan^{-1}(-1)$

Sol.

Let $\tan^{-1}(-1) = y$

$$\tan y = -1$$

$$\tan y = -\tan\left(\frac{\pi}{4}\right)$$

$$\tan y = \tan\left(-\frac{\pi}{4}\right)$$

$$y = -\frac{\pi}{4}$$

7. Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Sol.

Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$

$$\sec y = \frac{2}{\sqrt{3}}$$

$$\sec y = \sec\left(\frac{\pi}{6}\right)$$

$$y = \frac{\pi}{6}$$

8. Find the principal value of $\cot^{-1}\left(\sqrt{3}\right)$

Sol.

Let $\cot^{-1}\left(\sqrt{3}\right) = y$

$$\cot y = \sqrt{3}$$

$$\cot y = \cot\left(\frac{\pi}{6}\right)$$

$$y = \frac{\pi}{6}$$

9. Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Sol.

Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$

$$\cos y = -\frac{1}{\sqrt{2}}$$

$$\cos y = -\cos\left(\frac{\pi}{4}\right)$$

$$\cos y = \cos\left(\pi - \frac{\pi}{4}\right)$$

$$\cos y = \cos\left(\frac{3\pi}{4}\right)$$

$$y = \frac{3\pi}{4}$$

10. Find the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

Sol.

Let $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right) = y$

$$\operatorname{cosec} y = -\sqrt{2}$$

$$\operatorname{cosec} y = -\operatorname{cosec}\left(\frac{\pi}{4}\right)$$

$$\operatorname{cosec} y = \operatorname{cosec}\left(-\frac{\pi}{4}\right)$$

$$y = -\frac{\pi}{4}$$

11. Find the value of

$$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

Sol.

Let $\tan^{-1}(1) = A$

$$\tan A = 1$$

$$\tan A = \tan\frac{\pi}{4}$$

$$A = \frac{\pi}{4}$$

Let $\cos^{-1}\left(-\frac{1}{2}\right) = B$

$$\cos B = -\frac{1}{2}$$

$$\cos B = -\cos\left(\frac{\pi}{3}\right)$$

$$\cos B = \cos\left(\pi - \frac{\pi}{3}\right)$$

$$\cos B = \cos\left(\frac{2\pi}{3}\right)$$

$$B = \frac{2\pi}{3}$$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = c$

$$\sin c = -\frac{1}{2}$$

$$\sin c = -\sin\left(\frac{\pi}{6}\right)$$

$$\sin c = \sin\left(-\frac{\pi}{6}\right)$$

$$c = -\frac{\pi}{6}$$

$$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$\frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

$$= \frac{3\pi + 8\pi - 2\pi}{12}$$

$$= \frac{9\pi}{12}$$

$$= \frac{3\pi}{4}$$

12. Find the value of

$$\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$$

Sol.

Let $\cos^{-1}\left(\frac{1}{2}\right)=A$

$$\cos A=\frac{1}{2}$$

$$\cos A=\cos\left(\frac{\pi}{3}\right)$$

$$A=\frac{\pi}{3}$$

Let $\sin^{-1}\left(\frac{1}{2}\right)=B$

$$\sin B=\frac{1}{2}$$

$$\sin B=\sin\left(\frac{\pi}{6}\right)$$

$$B=\frac{\pi}{6}$$

$$\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$$

$$\frac{\pi}{3}+2\left(\frac{\pi}{6}\right)$$

$$\frac{\pi}{3}+\frac{\pi}{3}$$

$$=\frac{2\pi}{3}$$

13. Find the value of if $\sin^{-1} x = y$, then

(A) $0 \le y \le \pi$

(B) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(C) $0 < y < \pi$

(D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Sol.

$$\sin^{-1}(x) = y$$

Range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ option is correct

14. Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to

(A) π

(B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

Sol.

Let $\tan^{-1}\sqrt{3} = A$

$$\tan A = \sqrt{3}$$

$$\tan A = \tan\frac{\pi}{3}$$

$$A = \frac{\pi}{3}$$

Let $\sec^{-1}(-2) = B$

$$\sec B = -2$$

$$\sec B = -\sec\left(\frac{\pi}{3}\right)$$

$$\sec B = \sec\left(\pi - \frac{\pi}{3}\right)$$

$$\sec B = \sec\left(\frac{2\pi}{3}\right)$$

$$B = \frac{2\pi}{3}$$

Now,

$$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$$

$$= \frac{\pi}{3} - \frac{2\pi}{3}$$

$$= -\frac{\pi}{3}$$

Exercise-2.2

1. Prove

$$3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

Sol.

Let $x = \sin\varphi$

R.H.S.

$$\sin^{-1}\left(3x - 4x^3\right)$$
$$= \sin^{-1}\left[3\sin\varphi - 4\sin^3\varphi\right]$$
$$= \sin^{-1}\left(\sin 3\varphi\right)$$
$$= 3\varphi$$
$$= 3\sin^{-1}(x)$$

=L.H.S

2. Prove

$$3\cos^{-1}x = \cos^{-1}\left(4x^3 - 3x\right), x \in \left[\frac{1}{2}, 1\right]$$

Sol.

Let $x = \cos\varphi$

Then, $\varphi = \cos^{-1}x$

$$RHS = \cos^{-1}\left[4x^3 - 3x\right]$$

3. Prove

$$\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$$

Sol.

$$LHS=\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}$$

We know that, $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\dfrac{x+y}{1-xy}$

$$=\tan^{-1}\left[\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}.\frac{7}{24}}\right]$$

$$=\tan^{-1}\left[\frac{48+77}{264-14}\right]$$

$$=\tan^{-1}\left[\frac{125}{250}\right]$$

$$=\tan^{-1}\frac{1}{2}$$

$$=RHS$$

4. Prove

$$2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$$

Sol.

We know that, $2\tan^{-1}x=\tan^{-1}\left[\dfrac{2x}{1-x^2}\right]$

$$LHS=2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left[\frac{2.\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}\right] + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

Also, $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

$$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left[\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}.\frac{1}{7}}\right]$$

$$= \tan^{-1}\left[\frac{28+3}{21-4}\right]$$

$$= \tan^{-1}\left(\frac{31}{17}\right)$$

$$= RHS$$

5. Write the function in the simplest form:

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}, x \neq 0$$

Sol.

Let $x = \tan\theta$

$\theta = \tan^{-1}x$

$$\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}\right]$$

$$= \tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right]$$

$$= \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$= \tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right]$$

$$= \tan^{-1}\left[\tan\frac{\theta}{2}\right]$$

$$= \frac{\theta}{2}$$

$$= \frac{\tan^{-1}x}{2}$$

6. Write the function in the simplest form:

$$\tan^{-1}\frac{1}{\sqrt{x^2-1}}, |x| > 1$$

Sol.

Let $x = \operatorname{cosec} \theta$

$\theta = \operatorname{cosec}^{-1} x$

$$\tan^{-1} \frac{1}{\sqrt{x^2 - 1}}$$
$$= \tan^{-1} \frac{1}{\sqrt{\operatorname{cosec}^2 \theta - 1}}$$
$$= \tan^{-1} \frac{1}{\sqrt{\cot^2 \theta}}$$
$$= \tan^{-1} \left(\frac{1}{\cot \theta}\right)$$
$$= \tan^{-1} (\tan \theta)$$
$$= \theta$$
$$= \operatorname{cosec}^{-1} x$$

We know that

$$\left[\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}\right]$$

$$= \frac{\pi}{2} - \sec^{-1} x$$

7. Write the function in the simplest form:

$$\tan^{-1}\left[\sqrt{\frac{1 - \cos x}{1 + \cos x}}\right], x < \pi$$

Sol.

$$\tan^{-1}\left[\sqrt{\frac{1-\cos x}{1+\cos x}}\right]$$

$$=\tan^{-1}\left[\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right]$$

$$=\tan^{-1}\left[\frac{2\sin\frac{x}{2}}{2\cos\frac{x}{2}}\right]$$

$$=\tan^{-1}\left[\tan\frac{x}{2}\right]$$

$$=\frac{x}{2}$$

8. **Write the function in the simplest form:**

$$\tan^{-1}\left[\frac{\cos x-\sin x}{\cos x+\sin x}\right], 0<x<\pi$$

Sol.

$$\tan^{-1}\left[\frac{\cos x-\sin x}{\cos x+\sin x}\right]$$

divide by cos x

$$= \tan^{-1}\left[\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\right]$$

$$= \tan^{-1}\left[\frac{1-\tan x}{1+\tan x}\right]$$

$$= \tan^{-1}(1) - \tan^{-1}(\tan x)$$

We know that,

$$\tan^{-1}\frac{x-y}{1-xy} = \tan^{-1}x - \tan^{-1}y$$

$$= \frac{\pi}{4} - x$$

9. Write the function in the simplest form:

$$\tan^{-1}\frac{x}{\sqrt{a^2 - x^2}}, |x| < a$$

Sol.

Let $x = a\sin\theta$

$$\theta = \sin^{-1}\frac{x}{a}$$

$$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$$

$$=\tan^{-1}\frac{a\sin\theta}{\sqrt{a^2-(a\sin\theta)^2}}$$

$$=\tan^{-1}\frac{a\sin\theta}{\sqrt{a^2(1-\sin^2\theta)}}$$

$$=\tan^{-1}\frac{a\sin\theta}{\sqrt{a^2\cos^2\theta}}$$

$$=\tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$$

$$==\tan^{-1}(\tan\theta)$$

$$=\theta$$

$$=\sin^{-1}\frac{x}{a}$$

10. Write the function in the simplest form:

$$\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right), a>0;\ \frac{-a}{\sqrt{3}}\le x\le\frac{a}{\sqrt{3}}$$

Sol.

Let $x=a\tan\theta$

$$\theta=\tan^{-1}\left(\frac{x}{a}\right)$$

$$\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right)$$

$$= \tan^{-1}\left(\frac{3a^2(a\tan\theta) - (a\tan\theta)^3}{a^3 - 3a(a\tan\theta)^2}\right)$$

$$= \tan^{-1}\left(\frac{3a^3(\tan\theta) - a^3\tan^3\theta}{a^3 - 3a^3\tan^2\theta}\right)$$

$$= \tan^{-1}\left(\frac{3a^3(\tan\theta) - a^3\tan^3\theta}{a^3 - 3a^3\tan^2\theta}\right)$$

$$= \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 3\theta)$$

$$= 3\theta$$

$$= 3\tan^{-1}\left(\frac{x}{a}\right)$$

11. **Find the value of**

$$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$$

Sol.

$$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$$

$$=\tan^{-1}\left[2\cos\left\{2\sin^{-1}\left(\sin\frac{\pi}{6}\right)\right\}\right]$$

$$=\tan^{-1}\left[2\cos 2\left(\frac{\pi}{6}\right)\right]$$

$$=\tan^{-1}\left[2\cos\left(\frac{\pi}{3}\right)\right]$$

$$=\tan^{-1}\left[2\left(\frac{1}{2}\right)\right]$$

$$=\tan^{-1}[1]$$

$$=\tan^{-1}\left[\tan\frac{\pi}{4}\right]$$

$$=\frac{\pi}{4}$$

12. Find the value of

$$\mathbf{\cot\left(\tan^{-1}a+\cot^{-1}a\right)}$$

Sol.

$$\cot\left(\tan^{-1}a+\cot^{-1}a\right)$$

We know that $\left[\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right]$

$$=\cot\left(\frac{\pi}{2}\right)$$

$$=0$$

13. **Find the value of**

$$\tan\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right], |x|<1, y>0 \text{ and } xy<1$$

Sol.

Let $x = \tan\theta$

$\theta = \tan^{-1} x$

$y = \tan\varphi$

$\varphi = \tan^{-1} y$

$$\tan\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$$

$$= \tan\frac{1}{2}\left[\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)+\cos^{-1}\left(\frac{1-\tan^2\varphi}{1+\tan^2\varphi}\right)\right]$$

$$= \tan\frac{1}{2}\left[\sin^{-1}(\sin 2\theta)+\cos^{-1}(\cos 2\phi)\right]$$

$$= \tan\frac{1}{2}[2\theta+2\phi]$$

$$= \tan[\theta+\phi]$$

$$= \tan\left[\tan^{-1}x+\tan^{-1}y\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \frac{x+y}{1-xy}$$

14. **If** $\sin\left[\sin^{-1}\frac{1}{5}+\cos^{-1}x\right]=1$,

Then find the value of *x*.

Sol.

We know that

$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$

$$\sin\left[\sin^{-1}\frac{1}{5}+\cos^{-1}x\right]=1$$

$$=\sin\left(\sin^{-1}\frac{1}{5}\right)\cos\left(\cos^{-1}x\right)+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1$$

$$=\frac{1}{5}\times x+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1$$

$$=\frac{x}{5}+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1$$

$$Let\ \sin^{-1}\frac{1}{5}=y$$

$$\sin y=\frac{1}{5}$$

$$\cos y=\sqrt{1-\left(\frac{1}{5}\right)^2}=\frac{2\sqrt{6}}{5}$$

$$y=\cos^{-1}\left(\frac{2\sqrt{6}}{5}\right)$$

now,

$$\frac{x}{5}+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1$$

$$\frac{x}{5}+\cos y\sin\left(\cos^{-1}x\right)=1$$

$$\frac{x}{5}+\cos\left[\cos^{-1}\left(\frac{2\sqrt{6}}{5}\right)\right]\sin\left(\cos^{-1}x\right)=1$$

$$\frac{x}{5}+\frac{2\sqrt{6}}{5}\sin\left(\cos^{-1}x\right)=1$$

$$Let\ \ \cos^{-1}x=A$$

$$\cos A=x$$

$$\sin A=\sqrt{1-x^2}$$

$$A=\sin^{-1}\sqrt{1-x^2}$$

Now,

$$\frac{x}{5}+\frac{2\sqrt{6}}{5}\sin\left(\cos^{-1}x\right)=1$$

$$\frac{x}{5}+\frac{2\sqrt{6}}{5}\sin A=1$$

$$\frac{x}{5}+\frac{2\sqrt{6}}{5}\sin\left[\sin^{-1}\sqrt{1-x^2}\right]=1$$

$$\frac{x}{5}+\frac{2\sqrt{6}}{5}\left[\sqrt{1-x^2}\right]=1$$

$$x+2\sqrt{6}\left(\sqrt{1-x^2}\right)=5$$

$$2\sqrt{6}\left(\sqrt{1-x^2}\right)=5-x$$

On squaring both sides

$$\left(2\sqrt{6}\right)^2\left(\sqrt{1-x^2}\right)^2=\left(5-x\right)^2$$

$$24\left(1-x^2\right)=25+x^2-10x$$
$$24-24x^2=25+x^2-10x$$
$$25x^2-10x+1=0$$
$$\left(5x-1\right)^2=0$$
$$x=\frac{1}{5}$$

15. **If** $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$**, then find the value of** ***x*****.**

Sol.

We know that,

$$\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$$

$$\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$$

$$\tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right]=\frac{\pi}{4}$$

$$\tan^{-1}\left[\frac{(x-1)(x+2)+(x+1)(x-2)}{(x-2)(x+2)-(x-1)(x+1)}\right]=\frac{\pi}{4}$$

$$\tan^{-1}\left[\frac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1}\right]=\frac{\pi}{4}$$

$$\tan^{-1}\left[\frac{2x^2-4}{-3}\right]=\frac{\pi}{4}$$

$$\tan\frac{\pi}{4}=\frac{2x^2-4}{-3}$$

$$\frac{2x^2-4}{-3}=1$$

$$2x^2-4=-3$$

$$2x^2=1$$

$$x=\pm\frac{1}{\sqrt{2}}$$

16. Find the values of

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$$

Sol.

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$$

$$=\sin^{-1}\left[\sin\left(\pi-\frac{2\pi}{3}\right)\right]$$

$$=\sin^{-1}\left(\sin\frac{\pi}{3}\right)$$

$$=\frac{\pi}{3}$$

17. Find the values of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

Sol.

$$\tan^{-1}\left[\tan\frac{3\pi}{4}\right]$$

$$= \tan^{-1}\left[-\tan\left(\frac{-3\pi}{4}\right)\right]$$

$$= \tan^{-1}\left[-\tan\left(\pi-\frac{\pi}{4}\right)\right]$$

$$= \tan^{-1}\left[-\tan\left(\frac{\pi}{4}\right)\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{-\pi}{4}\right)\right]$$

$$= \frac{-\pi}{4}$$

18. Find the values of

$$\tan\left[\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right]$$

Sol.

$$\text{Let } \sin^{-1}\frac{3}{5} = x$$

$$\sin x = \frac{3}{5}$$

$$\cos x = \sqrt{1-\left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

$$\sec x = \frac{5}{4}$$

$$\therefore \tan x = \frac{3}{4}$$

$$x = \tan^{-1}\left(\frac{3}{4}\right)$$

$$\sin^{-1}\frac{3}{5} = \tan^{-1}\left(\frac{3}{4}\right)$$

Similarly,

$$\cot^{-1}\frac{3}{2} = \tan^{-1}\left(\frac{2}{3}\right)$$

Now,

$$\tan\left[\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$$

We know that,

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

$$= \tan\left[\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right]$$

$$= \tan\left[\tan^{-1}\frac{17}{12-6}\right]$$

$$= \frac{17}{6}$$

19. Find the values of

$\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ **is equal to**

(A) $\frac{7\pi}{6}$

(B) $\frac{5\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

Sol.

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$$

$$= \cos^{-1}\left(\cos\frac{-7\pi}{6}\right)$$

$$= \cos^{-1}\left[\cos\left(2\pi-\frac{7\pi}{6}\right)\right]$$

$$= \cos^{-1}\left[\cos\left(\frac{5\pi}{6}\right)\right]$$

$$= \frac{5\pi}{6}$$

The correct answer is B

20. Find the values of

$$\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$$ **is equal to**

(A) 1/2

(B) 1/3

(C) 1/4

(D) 1

Sol.

Let $\sin^{-1}\left(-\frac{1}{2}\right) = x$

$$\sin x = -\frac{1}{2}$$

$$\sin x = -\sin\left(\frac{\pi}{6}\right)$$

$$\sin x = \sin\left(-\frac{\pi}{6}\right)$$

$$x = -\frac{\pi}{6}$$

Now,

$$\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$$

$$= \sin\left[\frac{\pi}{3} - x\right]$$

$$= \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right]$$

$$= \sin\left[\frac{\pi}{2}\right]$$

$$= 1$$

The correct answer is D

21. Find the values of

$\tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right)$ is equal to

(A) π

(B) $-\frac{\pi}{2}$

(C) 0

(D) $2\sqrt{3}$

Sol.

Let $\cot^{-1}\left(-\sqrt{3}\right) = x$

$\cot x = -\sqrt{3}$

$\cot x = -\cot\left(\frac{\pi}{6}\right)$

$\cot x = \cot\left(\pi - \frac{\pi}{6}\right)$

$\cot x = \cot\left(\frac{5\pi}{6}\right)$

$x = \frac{5\pi}{6}$

Now,

$\tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right)$

$= \tan^{-1}\sqrt{3} - x$

$= \frac{\pi}{3} - \frac{5\pi}{6}$

$= -\frac{\pi}{2}$

The correct answer is B