Chapter -13
Nuclei
Class – XII
Subject – Physics

13.1. (a) Two stable isotopes of lithium 6 3Li and 7 3Li have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.
(b) Boron has two stable isotopes, 10 5B and 11 5B. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of 10 5B and 11 5 B.

Sol.

(a) Given:

Abundance of 3Li6 = 7.5%
Abundance of 3Li7 = 92.5%
Mass of 3Li6 = 6.01512 u
Mass of 3Li7 = 7.016 u

The atomic mass of lithium will depend on the atomic masses of these two isotopes with their abundances as given. This can be calculated by finding the weighted average of the two, as done below

Atomic mass of Li

= (6.01512 x 7.5 + 7.016 x 92.5) / 100
= 6.940934 u

= **6.941 u**

(b) Given:

Mass of 5B10 = 10.01294 u

Mass of 5B11 = 11.00931 u

Atomic mass of B = 10.811 u

The abundances of 5B10 and 5B11 can be calculated by using the technique employed in solution of part (a) of this question.

Let the abundance of 5B10 be y%

Then, the abundance of 5B11 = (100 – y)%

Now calculating weighted average

Atomic mass of boron = [10.01294y + 11.00931(100 – y)] / 100

Or

$$1081.1 = 10.01294y + 1100.931 - 11.00931y$$

Simplifying this linear eq. and solving further

$$0.99636y = 19.831$$

which gives

$$y = 19.9\%$$

$$100 - y = 80.1\%$$

Thus, abundance of 5B10 = 19.9%

And, abundance of 5B11 = 80.1%

13.2. The three stable isotopes of neon: 10Ne20, 10Ne21 and 10Ne22 have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

Sol.

Given:
Respective masses and abundances of three isotopes of neon.

Average atomic mass of Ne

$$= \frac{19.99 \times 90.51 + 20.99 \times 0.27 + 21.99 \times 9.22}{100}$$

= 20.1771 u

13.3. Obtain the binding energy (in MeV) of a nitrogen nucleus (14)7N, given m (14)7N =14.00307 u

Sol.

Binding energy of 7N14 = ΔMc^2
Mass defect = [mass of 7 protons + mass of 7 neutrons – actual mass of N]
= [14.11543 – 14.00307]
= 0.11236 u
Binding energy = 0.11236 u x 931.5 MeV / u
= **104.66 MeV**

13.4. Obtain the binding energy of the nuclei 56 26Fe and 209 83 Bi in units of MeV from the following data:
m (56 26Fe) = 55.934939 u
m (209 83 Bi) = 208.980388 u

Sol.

Mass defect of Fe = Theoretical value – Practical value
Solving as per previous question
Mass defect of Fe = 0.528461 u
BE of Fe = 0.528461 x 931.5
= 492.26 MeV
BE per nucleon of Fe = 492.26 / 56
= **8.97 MeV**

Similarly,
Mass defect of Bi = 1.760877 u
BE of Bi = 1640.257 MeV
BE per nucleon of Bi = 1640.257 / 209
= **7.84 MeV**

13.5. A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of 63 29Cu atoms (of mass 62.92960 u).

Sol.

Given:

Mass of coin = 0.003 kg

Mass of 29Cu63 = 62.9296 u

$= 1.045261 \times 10^{-25}$ kg

No. of Cu atoms in given sample

D = 0.003 / 1.045261×10^{-25}

$= 2.8701 \times 10^{22}$ atoms

Mass defect of 1 Cu atom = 0.591935

BE of 1 Cu atom = 0.591935 x 931.5 = 551.387 MeV

So,

BE of D atoms = $2.8701 \times 10^{22} \times 551.387$

= 1.583 x 10^{25} MeV

= 2.53 x 10^{12} J

13.6. Write nuclear reaction equations for

(i) α-decay of 226 88 Ra

(ii) α-decay of 242 94 Pu

(iii) β–-decay of 32 15 P

(iv) β–-decay of 210 83 Bi

(v) β+-decay of 11 6 C

(vi) β+-decay of 97 43 Tc

(vii) Electron capture of 120 54 Xe

Sol.

(i) α-decay of 226 88 Ra

$$_{88}Ra^{226} \rightarrow {}_{86}Rn^{222} + {}_{2}He^{4}$$

(ii) α-decay of 242 94 Pu

$$_{94}Pu^{242} \rightarrow {}_{92}U^{238} + {}_{2}He^{4}$$

(iii) β^{-}-decay of 32 15 P

$$_{15}P^{32} \rightarrow {}_{16}S^{32} + e^{-} + \bar{\nu}$$

(iv) β^{-}-decay of 210 83 Bi

$$_{83}Bi^{210} \rightarrow {}_{84}Po^{210} + e^{-} + \bar{\nu}$$

(v) β^{+}-decay of 11 6 C

$$_{6}C^{11} \rightarrow {}_{5}B^{11} + e^{+} + v$$

(vi) β^{+}-decay of 97 43 Tc

$$_{43}Tc^{97} \rightarrow {}_{42}Mo^{97} + e^{+} + v$$

(vii) Electron capture of 120 54 Xe

$${}_{54}Xe^{120} + e^{-} \rightarrow {}_{53}I^{120} + v$$

13.7. A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to
a) 3.125%,
b) 1% of its original value?

Sol.

Given:
$T_{1/2}$ = T years
$\lambda = 0.693 / T$

(a) N = 0.03125
$N_o = 1$
Using the expression
$$\ln N - \ln N_o = -\lambda t$$
Substituting values and solving
$$-3.466 = [-0.693 / T].t$$
Or **t = 5T years**

(b) N = 0.01
$N_o = 1$
Similarly

$\ln N - \ln N_o = -\lambda t$

Substituting values and solving

t = 6.65T years

13.8. The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive 14 6C present with the stable carbon isotope 12 6C . When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of 14 6C , and the measured activity, the age of the specimen can be approximately estimated. This is the principle of 14 6C dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Sol.

Given:

Initial Normal activity R = 15 decays / minute

= 0.25 decay / s

T-1/2 = 5730 years

= 1.807×10^{11} s

Therefore

Disintegration constant

$\lambda = 0.693 / T_{1/2}$

$= 0.693 / 1.807 \times 10^{11}$
$= 3.835 \times 10^{-12}$

Initial N_o

$R = \lambda N_o$

Or $N_o = 0.25 / \lambda$
$= 6.52 \times 10^{10}$ atoms

Specimen activity R' = 9 decays / minute
= 0.15 decay / s

Therefore N of specimen

$R' = \lambda N$

Or $N = 0.15 / 3.835 \times 10^{-12}$
$= 3.91 \times 10^{10}$ atoms

By law of radioactive decay

$\ln N - \ln N_o = -\lambda t$

Or

$t = (24.39 - 24.9) / (-3.835 \times 10^{-12})$
$= 1.332 \times 10^{11}$ s
= 4224 years

Thus the approximate age of Indus-Valley Civilization from the given sample is **4224 years**.

13.9. Obtain the amount of 60 27Co necessary to provide a radioactive source of 8.0 mCi strength. The half-life of 60 27Co is 5.3 years.

Sol.

Given:

R = 296000000 Bq

T-1/2 = 167162000 s

λ = 0.693 / T-1/2

= 4.146×10^{-9}

Therefore N = R / λ = 7.14×10^{16} atoms

Now

1 g of 27Co60 contains

= $[6.025 \times 10^{23}] / 60$

= 1.0037×10^{22} atoms

Hence

7.14×10^{16} atoms will be in

= $[7.14 \times 10^{16}] / [1.0037 \times 10^{22}]$

= $\mathbf{7.126 \times 10^{-6}}$ **g**

13.10. The half-life of 9038Sr is 28 years. What is the disintegration rate of 15 mg of this isotope?

Sol.

Disintegration rate, R = λN

λ = 0.693 / T-1/2
= 7.85 x 10^{-10}

15 g of Sr has
N = Avogadro's No. / 90
= 10^{20} atoms

Therefore R = (7.85 x 10^{-10}) x (10^{20})
= 7.879 x 10^{10} Bq
= 2.13 Ci

13.11. Obtain approximately the ratio of the nuclear radii of the gold isotope 197 79 Au and the silver isotope 107 47 Ag .

Sol.

Required ratio = Radius of Au isotope / Radius of Ag isotope

$$= \frac{R_0 \sqrt[3]{197}}{R_0 \sqrt[3]{107}}$$

$$= \frac{5.819}{4.747}$$

= 1.23

13.12. Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of

(a) 226 88 Ra and
(b) 220 86 Rn
Given m (226 88 Ra) = 226.02540 u, m (222 86 Rn) = 222.01750 u,
m (222 86 Rn) = 220.01137 u, m (216 84 Po) = 216.00189 u.

Sol.

(a) Nuclear reaction:

88Ra226 → 86Rn222 + 2He4

$$Q = (M_{Ra} - M_{Rn} - M_{He}).c^2$$
$$= (5.297 \times 10^{-3}).(931.5)$$
$$= \mathbf{4.934\ MeV}$$

Kinetic energy of emitted alpha particle:

The energy liberated in the nuclear reaction displays in the form of kinetic energy of the particles of products.

Kinetic energy of products

$$(MV^2 / 2) + (mv^2 / 2) = 4.93 \text{ MeV}\ldots\ldots\ldots\ldots(1)$$

where
M = mass of radon nuclei
m = mass of helium nuclei
V = velocity of radon nuclei
v = velocity of helium nuclei

To keep the momentum conserve

MV = mv

Or

$M / m = v / V$……………………………(2)

This implies nothing but the simple fact that the most of the kinetic energy (or Q calculated above) will be retained by the helium nuclei, eq. (2) pointing that it will travel with much large velocity as compared to the velocity of the radon nuclei.
Dividing eq. (1) throughout by V^2 and making substitutions from eq. (2)

$(M^2 / m) + M = 9.86 / V^2$
Or
$V^2 = 7.86 \times 10^{-4}$
KE of Rn = $MV^2 / 2 = 0.087$ MeV
therefore
KE of alpha particle = 4.93 – 0.087
= **4.85 MeV**

(b) Nuclear reaction:
$_{86}Rn^{220} \rightarrow {}_{84}Po^{216} + {}_{2}He^{4}$

Calculating Q in a similar fashion
Q = 6.41 MeV

Kinetic energy of emitted alpha particle:
As solved in part (a) of this question

KE of products
$[(MV)V / 2] + [(mv)v / 2] = 6.41$
where
M = mass of polonium nuclei
m = mass of helium nuclei

V = velocity of polonium nuclei
v = velocity of helium nuclei

By the same arguments and solving technique used in part (a) of this question, V comes to be

V = 0.033

KE of Po = $MV^2 / 2$
= 0.117 MeV

Therefore KE of alpha particle
= 6.41 – 0.117
= **6.29 MeV**

13.13. The radionuclide 11C decays according to
11 + 6C → 5 B+e +ν : T1/2=20.3 min
The maximum energy of the emitted positron is 0.960 MeV. Given the mass values:
m (11 6C) = 11.011434 u and m (11 6B) = 11.009305 u, calculate Q and compare it with the maximum energy of the positron emitted.

Sol.

Nuclear Reaction:
6C11 → 5B11 + e^+ v + Q

Q = [mass of C – mass of B – mass of electron – mass of electron] .c.c

= 0.961 MeV

This Q has energy distributed as

$Q = Ed + Ee + Ev$

For constant momentum, the energy carried by daughter nuclei and neutrino is nearly zero. The positron carries maximum energy.
Hence max $Ee \approx Q$.

13.14. The nucleus 23 10 Ne decays by β– emission. Write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:
m (23 10 Ne) = 22.994466 u
m (23 11 Na) = 22.089770 u.

Sol.

Nuclear Reaction:

$10Ne23 \rightarrow 11Na23 + e^- + \nu + Q$

Q = [mass of Ne – mass of Na – mass of electron].c.c
Using atomic masses

Q = [mass of Ne – mass of Ma].c.c
= 4.37 MeV

For the reasons argued in previous questions, the maximum kinetic energy of electron

Maximum Ee = Q = 4.37 MeV

13.15. The Q value of a nuclear reaction A + b → C + d is defined by

Q = [mA + mb – mC – md]c2

where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) 1H1+ 1H3 → 1H2+ 1H2

(ii) 6C12+ 6C12 → 10Ne20+ 2He4

Atomic masses are given to be

m (2 1H) = 2.014102 u

m (3 1H) = 3.016049 u

m (12 6C) = 12.000000 u

m (20 10 Ne) = 19.992439 u

Sol.

A + b → C + d

Q = [mass of A + mass of b – mass of C – mass of d].c.c

(i) Nuclear Reaction:

1H1 + 1H3 → 1H2 + 1H2

Q = [-4.33 X 10-3].[931.5]

= **-4.03 MeV**

Reaction is **endothermic**. This much energy has to be supplied.

(ii) Nuclear Reaction:

6C12 + 6C12 → 10Ne20 + 2He4

Q = [4.958 x 10-3].[931.5]

= **4.62 MeV**

Positive sign indicates the reaction is **exothermic**.

13.16. Suppose, we think of fission of a 56 26Fe nucleus into two equal fragments, 28 13 Al . Is the fission energetically possible? Argue by working out Q of the process. Given m (56 26Fe) = 55.93494 u and m (28 13 Al) = 27.98191 u.

Sol.

Nuclear Reaction:

26Fe56 → 13Al28 + 13Al28 + Q

Q = [mass of Fe – mass of aluminium – mass of aluminium].c.c

= [-0.0288].[931.5]

= **-26.9 MeV**

The reaction being endothermic is **not possible**.

13.17. The fission properties of 239 94 Pu are very similar to those of 235 92 U. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure 239 94 Pu undergo fission?

Sol.

Given:
Energy released per fission = 180 MeV

1 g of Pu contains
= Avogadro's No. / 239

1 kg of Pu contains
$N = 2.5197 \times 10^{24}$ atoms

1 atom releases = 180 MeV
Therefore,
N atoms release = 4.536×10^{26} MeV

13.18. A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much 235 92 U did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of 235 92 U and that this nuclide is consumed only by the fission process.

Sol.

Energy generated per gram of U
$= (6 \times 10^{23} \times 200 \times 1.6 \times 10^{-13}) / 235$

the amount of U consumed in 5 years with 80% on-time

$= (5 \times 0.8 \times 3.154 \times 10^{16} \times 235) / (1.2 \times 1.6 \times 10^{13})$
$= 1544$ kg

Thus the initial amount = **3088 kg**

13.19. How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as
$_1H^2 + _1H^2 \rightarrow _2He^3 + n + 3.27$ MeV

Sol.

Nuclear Reaction:
$_1H^2 + _1H^2 \rightarrow _2He^3 + n + 3.27$ MeV

No. of atoms in 2 kg deuterium
S = (Avogadro's No. x 2000) / 2
$= 6.023 \times 10^{26}$ atoms

Energy of 1 atom = 3.27 / 2
= 1.635 MeV

Therefore
Energy of S atoms $= 9.85 \times 10^{26}$ MeV
$= 1.576 \times 10^{14}$ J

We know
Power = Energy / Time

Hence,
Time $= 1.576 \times 10^{12}$ s

$= \textbf{About } \mathbf{4.9 \times 10^4} \textbf{ years}$

13.20. Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Sol.

Given:
R = 2 fm
For head on collision
d = R + R
= 4 fm

Height of the potential barrier

$$V = \frac{e^2}{4\pi\epsilon_0 d}$$

Substituting values yields
$V = 5.76 \times 10^{-14}$ J
= **360 keV**

13.21. From the relation R = R0A1/3, where R0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

Sol.

We know

Nuclear mass density = mass / volume……………(1)

Now Mass $\propto$ A

And Volume $\propto R^3$

Since $R \propto A^{1/3}$

Therefore

Volume $\propto$ A

When putting these proportional values in eq. (1), A gets cancelled. Hence shown!

13.22. For the β+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K–shell, is captured by the nucleus and a neutrino is emitted).

$e^+ + ZXA \rightarrow (Z-1)YA + v$

Show that if β+ emission is energetically allowed, electron capture is necessarily allowed but not vice–versa.

Sol.

The two processes:

for positron capture

$$_{Z}X^{A} \rightarrow {}_{Z-1}Y^{A} + e^{+} + v_{e} + Q_{1}$$

for electron capture

$$e^{-} + {}_{Z}X^{A} \rightarrow {}_{Z-1}Y^{A} + v_{e} + Q_{2}$$

Calculating Q_1

$$Q_1 = [M_N({}_{Z}X^{A}) - ZM_e - M({}_{Z-1}Y^{A}) - (Z-1)M_e - M_e].c^2$$
$$= [M({}_{Z}X^{A}) - M({}_{Z-1}Y^{A}) - 2M_e]c^2$$

Calculation for Q_2

$$Q_2 = [M_N({}_{Z}X^{A}) + M_e - M_N({}_{Z-1}Y^{A})]c^2$$
$$= [M({}_{Z}X^{A}) - M({}_{Z-1}Y^{A})]c^2$$

From the above calculations, it is obvious that

if $Q_1 > 0$

then $Q_2 > 0$

But if $Q_2 > 0$

then it can't be said that $Q_1 > 0$

Hence the result!