

Chapter -13
Nuclei
Class – XII
Subject – Physics

- 13.1.** (a) Two stable isotopes of lithium ${}^6_3\text{Li}$ and ${}^7_3\text{Li}$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.
- (b) Boron has two stable isotopes, ${}^{10}_5\text{B}$ and ${}^{11}_5\text{B}$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of ${}^{10}_5\text{B}$ and ${}^{11}_5\text{B}$.

Sol.

(a) Given:

Abundance of ${}^6_3\text{Li}$ = 7.5%

Abundance of ${}^7_3\text{Li}$ = 92.5%

Mass of ${}^6_3\text{Li}$ = 6.01512 u

Mass of ${}^7_3\text{Li}$ = 7.016 u

The atomic mass of lithium will depend on the atomic masses of these two isotopes with their abundances as given. This can be calculated by finding the weighted average of the two, as done below

Atomic mass of Li

$$= (6.01512 \times 7.5 + 7.016 \times 92.5) / 100$$

$$= 6.940934 \text{ u}$$

$$= 6.941 \text{ u}$$

(b) Given:

$$\text{Mass of } ^{10}\text{B} = 10.01294 \text{ u}$$

$$\text{Mass of } ^{11}\text{B} = 11.00931 \text{ u}$$

$$\text{Atomic mass of B} = 10.811 \text{ u}$$

The abundances of ^{10}B and ^{11}B can be calculated by using the technique employed in solution of part (a) of this question.

Let the abundance of ^{10}B be $y\%$

Then, the abundance of $^{11}\text{B} = (100 - y)\%$

Now calculating weighted average

$$\text{Atomic mass of boron} = [10.01294y + 11.00931(100 - y)] / 100$$

Or

$$1081.1 = 10.01294y + 1100.931 - 11.00931y$$

Simplifying this linear eq. and solving further

$$0.99636y = 19.831$$

which gives

$$y = 19.9\%$$

$$100 - y = 80.1\%$$

Thus, abundance of $^{10}\text{B} = 19.9\%$

And, abundance of $^{11}\text{B} = 80.1\%$

- 13.2.** The three stable isotopes of neon: ^{20}Ne , ^{21}Ne and ^{22}Ne have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the

average atomic mass of neon.

Sol.

Given:

Respective masses and abundances of three isotopes of neon.

Average atomic mass of Ne

$$= \frac{19.99 \times 90.51 + 20.99 \times 0.27 + 21.99 \times 9.22}{100}$$
$$= 20.1771 \text{ u}$$

- 13.3.** Obtain the binding energy (in MeV) of a nitrogen nucleus $(14)7\text{N}$, given $m(14)7\text{N} = 14.00307 \text{ u}$

Sol.

Binding energy of $7\text{N}14 = \Delta Mc^2$

Mass defect = [mass of 7 protons + mass of 7 neutrons – actual mass of N]

$$= [14.11543 - 14.00307]$$

$$= 0.11236 \text{ u}$$

Binding energy = $0.11236 \text{ u} \times 931.5 \text{ MeV / u}$

$$= 104.66 \text{ MeV}$$

13.4. Obtain the binding energy of the nuclei $^{56}_{26}\text{Fe}$ and $^{209}_{83}\text{Bi}$ in units of MeV from the following data:

$$m(^{56}_{26}\text{Fe}) = 55.934939 \text{ u}$$

$$m(^{209}_{83}\text{Bi}) = 208.980388 \text{ u}$$

Sol.

Mass defect of Fe = Theoretical value – Practical value

Solving as per previous question

$$\text{Mass defect of Fe} = 0.528461 \text{ u}$$

$$\begin{aligned}\text{BE of Fe} &= 0.528461 \times 931.5 \\ &= 492.26 \text{ MeV}\end{aligned}$$

$$\begin{aligned}\text{BE per nucleon of Fe} &= 492.26 / 56 \\ &= \mathbf{8.97 \text{ MeV}}\end{aligned}$$

Similarly,

$$\text{Mass defect of Bi} = 1.760877 \text{ u}$$

$$\text{BE of Bi} = 1640.257 \text{ MeV}$$

$$\begin{aligned}\text{BE per nucleon of Bi} &= 1640.257 / 209 \\ &= \mathbf{7.84 \text{ MeV}}\end{aligned}$$

13.5. A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}\text{Cu}$ atoms (of mass 62.92960 u).

Sol.

Given:

Mass of coin = 0.003 kg

Mass of $^{63}_{29}\text{Cu}$ = 62.9296 u
= 1.045261×10^{-25} kg

No. of Cu atoms in given sample

$$D = 0.003 / 1.045261 \times 10^{-25}$$
$$= 2.8701 \times 10^{22} \text{ atoms}$$

Mass defect of 1 Cu atom = 0.591935

BE of 1 Cu atom = $0.591935 \times 931.5 = 551.387$ MeV

So,

$$\text{BE of D atoms} = 2.8701 \times 10^{22} \times 551.387$$
$$= \mathbf{1.583 \times 10^{25} \text{ MeV}}$$
$$= \mathbf{2.53 \times 10^{12} \text{ J}}$$

13.6. Write nuclear reaction equations for

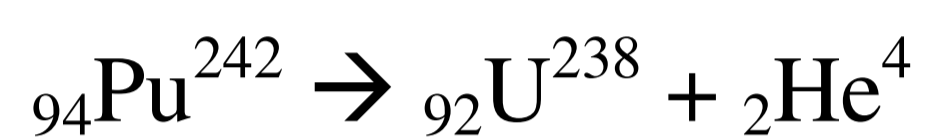
- (i) α -decay of $^{226}_{88}\text{Ra}$
- (ii) α -decay of $^{242}_{94}\text{Pu}$
- (iii) β^- -decay of $^{32}_{15}\text{P}$
- (iv) β^- -decay of $^{210}_{83}\text{Bi}$
- (v) β^+ -decay of $^{11}_6\text{C}$
- (vi) β^+ -decay of $^{97}_{43}\text{Tc}$
- (vii) Electron capture of $^{120}_{54}\text{Xe}$

Sol.

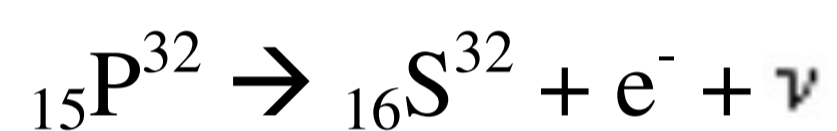
(i) α -decay of $^{226}_{88}\text{Ra}$



(ii) α -decay of $^{242}_{94}\text{Pu}$



(iii) β^- -decay of $^{32}_{15}\text{P}$



(iv) β^- -decay of $^{210}_{83}\text{Bi}$



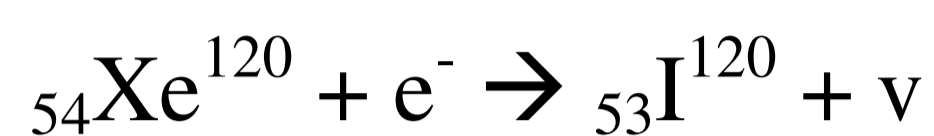
(v) β^+ -decay of $^{11}_6\text{C}$



(vi) β^+ -decay of $^{97}_{43}\text{Tc}$



(vii) Electron capture of $^{120}_{54}\text{Xe}$



13.7. A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to

- a) 3.125%,
- b) 1% of its original value?

Sol.

Given:

$$T_{1/2} = T \text{ years}$$

$$\lambda = 0.693 / T$$

(a) $N = 0.03125$

$$N_0 = 1$$

Using the expression

$$\ln N - \ln N_0 = -\lambda t$$

Substituting values and solving

$$-3.466 = [-0.693 / T].t$$

Or $t = 5T \text{ years}$

(b) $N = 0.01$

$$N_0 = 1$$

Similarly

$$\ln N - \ln N_0 = -\lambda t$$

Substituting values and solving

$$t = 6.65T \text{ years}$$

- 13.8.** The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive ^{14}C present with the stable carbon isotope ^{12}C . When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of ^{14}C , and the measured activity, the age of the specimen can be approximately estimated. This is the principle of ^{14}C dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Sol.

Given:

$$\begin{aligned}\text{Initial Normal activity } R &= 15 \text{ decays / minute} \\ &= 0.25 \text{ decay / s}\end{aligned}$$

$$\begin{aligned}T_{-1/2} &= 5730 \text{ years} \\ &= 1.807 \times 10^{11} \text{ s}\end{aligned}$$

Therefore

$$\begin{aligned}\text{Disintegration constant} \\ \lambda &= 0.693 / T_{-1/2}\end{aligned}$$

$$\begin{aligned} &= 0.693 / 1.807 \times 10^{11} \\ &= 3.835 \times 10^{-12} \end{aligned}$$

Initial N_0

$$R = \lambda N_0$$

Or
$$\begin{aligned} N_0 &= 0.25 / \lambda \\ &= 6.52 \times 10^{10} \text{ atoms} \end{aligned}$$

Specimen activity $R' = 9 \text{ decays / minute}$
 $= 0.15 \text{ decay / s}$

Therefore N of specimen

$$R' = \lambda N$$

Or
$$\begin{aligned} N &= 0.15 / 3.835 \times 10^{-12} \\ &= 3.91 \times 10^{10} \text{ atoms} \end{aligned}$$

By law of radioactive decay

$$\ln N - \ln N_0 = -\lambda t$$

Or

$$\begin{aligned} t &= (24.39 - 24.9) / (-3.835 \times 10^{-12}) \\ &= 1.332 \times 10^{11} \text{ s} \\ &= 4224 \text{ years} \end{aligned}$$

Thus the approximate age of Indus-Valley Civilization from the given sample is **4224 years**.

- 13.9.** Obtain the amount of $^{60}_{27}\text{Co}$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}\text{Co}$ is 5.3 years.

Sol.

Given:

$$R = 296000000 \text{ Bq}$$

$$T_{-1/2} = 167162000 \text{ s}$$

$$\lambda = 0.693 / T_{-1/2}$$

$$= 4.146 \times 10^{-9}$$

$$\text{Therefore } N = R / \lambda = 7.14 \times 10^{16} \text{ atoms}$$

Now

1 g of $^{27}\text{Co60}$ contains

$$= [6.025 \times 10^{23}] / 60$$

$$= 1.0037 \times 10^{22} \text{ atoms}$$

Hence

7.14×10^{16} atoms will be in

$$= [7.14 \times 10^{16}] / [1.0037 \times 10^{22}]$$

$$= \mathbf{7.126 \times 10^{-6} \text{ g}}$$

13.10. The half-life of ^{90}Sr is 28 years. What is the disintegration rate of 15 mg of this isotope?

Sol.

Disintegration rate, $R = \lambda N$

$$\lambda = 0.693 / T_{1/2}$$
$$= 7.85 \times 10^{-10}$$

15 g of Sr has

$$N = \text{Avogadro's No.} / 90$$
$$= 10^{20} \text{ atoms}$$

$$\text{Therefore } R = (7.85 \times 10^{-10}) \times (10^{20})$$
$$= 7.879 \times 10^{10} \text{ Bq}$$
$$= 2.13 \text{ Ci}$$

13.11. Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}\text{Au}$ and the silver isotope $^{107}_{47}\text{Ag}$.

Sol.

Required ratio = Radius of Au isotope / Radius of Ag isotope

$$= \frac{R_0 \sqrt[3]{197}}{R_0 \sqrt[3]{107}}$$

$$= \frac{5.819}{4.747}$$

$$= 1.23$$

13.12. Find the Q-value and the kinetic energy of the emitted α -particle in the α -decay of

(a) $^{226}_{88}\text{Ra}$ and

(b) $^{220}_{86}\text{Rn}$

Given $m(^{226}_{88}\text{Ra}) = 226.02540 \text{ u}$, $m(^{222}_{86}\text{Rn}) = 222.01750 \text{ u}$,
 $m(^{222}_{86}\text{Rn}) = 220.01137 \text{ u}$, $m(^{216}_{84}\text{Po}) = 216.00189 \text{ u}$.

Sol.

(a) Nuclear reaction:



$$\begin{aligned} Q &= (M_{\text{Ra}} - M_{\text{Rn}} - M_{\text{He}}).c^2 \\ &= (5.297 \times 10^{-3}).(931.5) \\ &= \mathbf{4.934 \text{ MeV}} \end{aligned}$$

Kinetic energy of emitted alpha particle:

The energy liberated in the nuclear reaction displays in the form of kinetic energy of the particles of products.

Kinetic energy of products

$$(MV^2 / 2) + (mv^2 / 2) = 4.93 \text{ MeV} \dots \dots \dots (1)$$

where

M = mass of radon nuclei

m = mass of helium nuclei

V = velocity of radon nuclei

v = velocity of helium nuclei

To keep the momentum conserve

$$MV = mv$$

Or

$$M / m = v / V \dots\dots\dots(2)$$

This implies nothing but the simple fact that the most of the kinetic energy (or Q calculated above) will be retained by the helium nuclei, eq. (2) pointing that it will travel with much large velocity as compared to the velocity of the radon nuclei.

Dividing eq. (1) throughout by V^2 and making substitutions from eq. (2)

$$(M^2 / m) + M = 9.86 / V^2$$

Or

$$V^2 = 7.86 \times 10^{-4}$$

$$\text{KE of Rn} = MV^2 / 2 = 0.087 \text{ MeV}$$

therefore

$$\begin{aligned} \text{KE of alpha particle} &= 4.93 - 0.087 \\ &= \mathbf{4.85 \text{ MeV}} \end{aligned}$$

(b) Nuclear reaction:



Calculating Q in a similar fashion

$$\mathbf{Q = 6.41 \text{ MeV}}$$

Kinetic energy of emitted alpha particle:

As solved in part (a) of this question

KE of products

$$[(MV)V / 2] + [(mv)v / 2] = 6.41$$

where

M = mass of polonium nuclei

m = mass of helium nuclei

V = velocity of polonium nuclei

v = velocity of helium nuclei

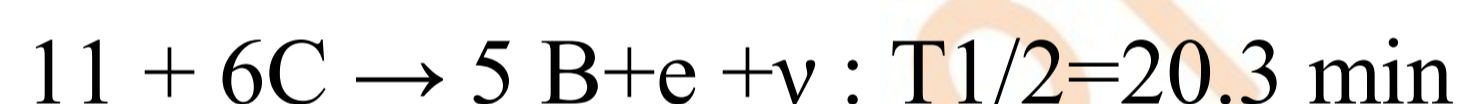
By the same arguments and solving technique used in part (a) of this question, V comes to be

$$V = 0.033$$

$$\begin{aligned}\text{KE of Po} &= MV^2 / 2 \\ &= 0.117 \text{ MeV}\end{aligned}$$

$$\begin{aligned}\text{Therefore KE of alpha particle} \\ &= 6.41 - 0.117 \\ &= \mathbf{6.29 \text{ MeV}}\end{aligned}$$

13.13. The radionuclide $^{11}_{6}\text{C}$ decays according to



The maximum energy of the emitted positron is 0.960 MeV. Given the mass values:

$m(^{11}_{6}\text{C}) = 11.011434 \text{ u}$ and $m(^{11}_{5}\text{B}) = 11.009305 \text{ u}$, calculate Q and compare it with the maximum energy of the positron emitted.

Sol.

Nuclear Reaction:



$$Q = [\text{mass of C} - \text{mass of B} - \text{mass of electron} - \text{mass of electron}] .c.c$$

$$= 0.961 \text{ MeV}$$

This Q has energy distributed as

$$Q = E_d + E_e + E_\nu$$

For constant momentum, the energy carried by daughter nuclei and neutrino is nearly zero. The positron carries maximum energy.

Hence $\max E_e \approx Q$.

13.14. The nucleus ${}^{23}_{10}\text{Ne}$ decays by β^- emission. Write down the β^- -decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$$m({}^{23}_{10}\text{Ne}) = 22.994466 \text{ u}$$

$$m({}^{23}_{11}\text{Na}) = 22.989770 \text{ u}.$$

Sol.

Nuclear Reaction:



$$Q = [\text{mass of Ne} - \text{mass of Na} - \text{mass of electron}]c^2$$

Using atomic masses

$$\begin{aligned} Q &= [\text{mass of Ne} - \text{mass of Na}]c^2 \\ &= 4.37 \text{ MeV} \end{aligned}$$

For the reasons argued in previous questions, the maximum kinetic energy

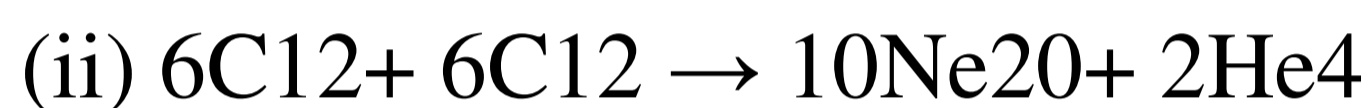
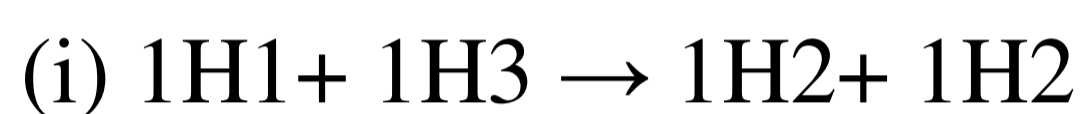
of electron

$$\text{Maximum } E_e = Q = 4.37 \text{ MeV}$$

13.15. The Q value of a nuclear reaction $A + b \rightarrow C + d$ is defined by

$$Q = [m_A + m_b - m_C - m_d]c^2$$

where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.



Atomic masses are given to be

$$m(2\text{H}) = 2.014102 \text{ u}$$

$$m(3\text{H}) = 3.016049 \text{ u}$$

$$m(12\text{C}) = 12.000000 \text{ u}$$

$$m(20\text{Ne}) = 19.992439 \text{ u}$$

Sol.



$$Q = [\text{mass of } A + \text{mass of } b - \text{mass of } C - \text{mass of } d].c^2$$

(i) Nuclear Reaction:



$$\begin{aligned} Q &= [-4.33 \times 10^{-3}][931.5] \\ &= -4.03 \text{ MeV} \end{aligned}$$

Reaction is **endothermic**. This much energy has to be supplied.

(ii) Nuclear Reaction:



$$Q = [4.958 \times 10^{-3}] \cdot [931.5] \\ = \mathbf{4.62 \text{ MeV}}$$

Positive sign indicates the reaction is **exothermic**.

13.16. Suppose, we think of fission of a $^{56}_{26}\text{Fe}$ nucleus into two equal fragments, $^{28}_{13}\text{Al}$. Is the fission energetically possible? Argue by working out Q of the process. Given $m(^{56}_{26}\text{Fe}) = 55.93494 \text{ u}$ and $m(^{28}_{13}\text{Al}) = 27.98191 \text{ u}$.

Sol.

Nuclear Reaction:



$$Q = [\text{mass of Fe} - \text{mass of aluminium} - \text{mass of aluminium}] \cdot c \cdot c \\ = [-0.0288] \cdot [931.5] \\ = \mathbf{-26.9 \text{ MeV}}$$

The reaction being endothermic is **not possible**.

13.17. The fission properties of $^{239}_{94}\text{Pu}$ are very similar to those of $^{235}_{92}\text{U}$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure $^{239}_{94}\text{Pu}$ undergo fission?

Sol.

Given:

Energy released per fission = 180 MeV

1 g of Pu contains
= Avogadro's No. / 239

1 kg of Pu contains
 $N = 2.5197 \times 10^{24}$ atoms

1 atom releases = 180 MeV

Therefore,

N atoms release = 4.536×10^{26} MeV

13.18. A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much $^{235}_{92}\text{U}$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of $^{235}_{92}\text{U}$ and that this nuclide is consumed only by the fission process.

Sol.

Energy generated per gram of U
 $= (6 \times 10^{23} \times 200 \times 1.6 \times 10^{-13}) / 235$

the amount of U consumed in 5 years with 80% on-time

$$\begin{aligned} &= (5 \times 0.8 \times 3.154 \times 10^{16} \times 235) / (1.2 \times 1.6 \times 10^{13}) \\ &= 1544 \text{ kg} \end{aligned}$$

Thus the initial amount = **3088 kg**

13.19. How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as
 $1\text{H}_2 + 1\text{H}_2 \rightarrow 2\text{He}_3 + n + 3.27 \text{ MeV}$

Sol.

Nuclear Reaction:



No. of atoms in 2 kg deuterium

$$\begin{aligned} S &= (\text{Avogadro's No.} \times 2000) / 2 \\ &= 6.023 \times 10^{26} \text{ atoms} \end{aligned}$$

$$\begin{aligned} \text{Energy of 1 atom} &= 3.27 / 2 \\ &= 1.635 \text{ MeV} \end{aligned}$$

Therefore

$$\begin{aligned} \text{Energy of } S \text{ atoms} &= 9.85 \times 10^{26} \text{ MeV} \\ &= 1.576 \times 10^{14} \text{ J} \end{aligned}$$

We know

$$\text{Power} = \text{Energy} / \text{Time}$$

Hence,

$$\text{Time} = 1.576 \times 10^{12} \text{ s}$$

$$= \text{About } 4.9 \times 10^4 \text{ years}$$

13.20. Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Sol.

Given:

$$R = 2 \text{ fm}$$

For head on collision

$$\begin{aligned} d &= R + R \\ &= 4 \text{ fm} \end{aligned}$$

Height of the potential barrier

$$V = \frac{e^2}{4\pi\epsilon_0 d}$$

Substituting values yields

$$\begin{aligned} V &= 5.76 \times 10^{-14} \text{ J} \\ &= \mathbf{360 \text{ keV}} \end{aligned}$$

13.21. From the relation $R = R_0 A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

Sol.

We know

$$\text{Nuclear mass density} = \text{mass} / \text{volume} \dots \dots \dots (1)$$

$$\text{Now} \quad \text{Mass} \propto A$$

$$\text{And} \quad \text{Volume} \propto R^3$$

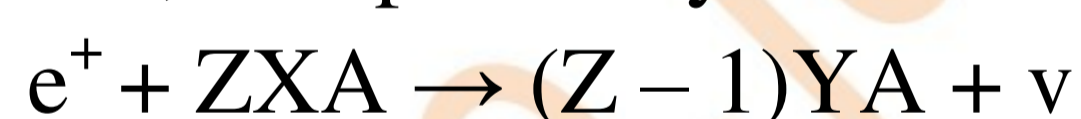
$$\text{Since} \quad R \propto A^{1/3}$$

Therefore

$$\text{Volume} \propto A$$

When putting these proportional values in eq. (1), A gets cancelled. Hence shown!

13.22. For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K-shell, is captured by the nucleus and a neutrino is emitted).

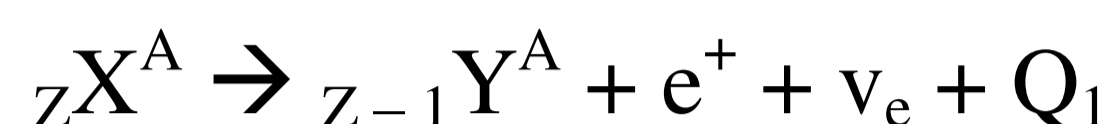


Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice-versa.

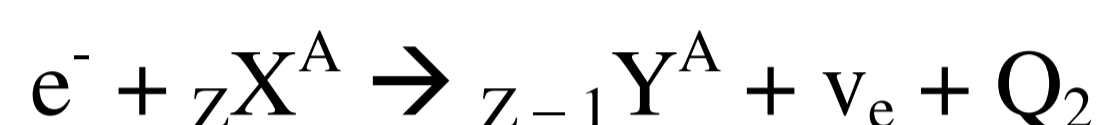
Sol.

The two processes:

for positron capture



for electron capture



Calculating Q_1

$$\begin{aligned} Q_1 &= [M_N({}_Z X^A) - ZM_e - M({}_{Z-1} Y^A) - (Z-1)M_e - M_e] \cdot c^2 \\ &= [M({}_Z X^A) - M({}_{Z-1} Y^A) - 2M_e] c^2 \end{aligned}$$

Calculation for Q_2

$$\begin{aligned} Q_2 &= [M_N({}_Z X^A) + M_e - M_N({}_{Z-1} Y^A)] c^2 \\ &= [M({}_Z X^A) - M({}_{Z-1} Y^A)] c^2 \end{aligned}$$

From the above calculations, it is obvious that

if $Q_1 > 0$
then $Q_2 > 0$

But if $Q_2 > 0$
then it can't be said that $Q_1 > 0$

Hence the result!