Chapter -11
Dual Nature of Radiation and Matter
Class – XII
Subject – Physics

11.1. Find the

a) maximum frequency, and

b) Minimum wavelength of X-rays produced by 30 kV electrons.

Sol.

a) We know,

$K_{max} = hv - \Phi_o$

Or $eV_o = hv - \Phi_o$

$v_{max} = eV_o / h$

Substituting the required values

$v_{max} = (1.6 \times 10^{-19}).(30000) / (6.63 \times 10^{-34})$

$= 7.24 \times 10^{18}$ Hz

b) By the relation

$\lambda = c / v_{max}$

$= 0.041$ nm

11.2. The work function of caesium metal is 2.14 eV. When light of frequency 6×10^{14}Hz is incident on the metal surface, photoemission of electrons occurs.

a) What is the maximum kinetic energy of the emitted electrons,

b) Stopping potential, and
c) Maximum speed of the emitted photoelectrons?

Sol.

Given:

$\Phi_o = 2.14$ V

$v = 6 \times 10^{14}$ Hz

a) Maximum kinetic energy is the differnce between the photon energy and the work function of the metal. For caesium it is given to be 2.14 V.

Using the expression

$K_{max} = hv - \Phi_o$

Substitution yields

$K_{max} = [(6.63 \times 10^{-34}).(6 \times 10^{14})] - [(2.14).(1.6 \times 10^{-19})]$

$= 5.54 \times 10^{-20}$ J

$= 0.346$ eV

b) Stopping potential can be deduced by maximum kinetic energy,

by the expression

eVo = Kmax

Or Vo = Kmax / e

Putting the required values

$Vo = 5.54 \times 10^{-20} / 1.6 \times 10^{-19}$

$= 0.34$ V

c) Maximum speed of photoelectron is given by the maximum kinetic energy it possesses,

$Kmax = 5.54 \times 10^{-20}$

Or $mv^2 / 2 = 5.54 \times 10^{-20}$

Or $v_{max} = 344$ km / s

11.3. The photoelectric cut-off voltage in a certain experiment is 1.5 V.What is the maximum kinetic energy of photoelectrons emitted?

Sol.

Maximum kinetic energy,
Kmax = eVo
Putting values and solving
$= 2.4 \times 10^{-19}$ J

11.4. Monochromatic light of wavelength 632.8 nm is produced by ahelium-neon laser. The power emitted is 9.42 mW.

a) Find the energy and momentum of each photon in the light beam,
b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and
c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

Sol.

Given:
Wavelength of monochromatic light
$\lambda = 632.8$ nm
Power of He-Ne laser

$$P = 9.42 \text{ mW}$$

a) Energy of a photon is given by

$$E = h\nu$$

Or $$E = hc / \lambda$$

which gives

$E = 3.14 \times 10^{-19}$ J

Now momentum of a photon

$$p = h / \lambda$$

Substitution yields

$p = 1.05 \times 10^{-27}$ kg m / s

b) For a beam of uniform cross-section having cross-sectional area less than target area

$P = E \times N$

where

P = power emitted

E = energy of photon

N = number of photons

Therefore

$$N = P / E$$

Substitution gives

$N = 3 \times 10^{16}$ photons / second

c) Momentum of He-Ne laser = 1.05×10^{-27} kg m / s

For this much momentum of a hydrogen atom

$$mv = 1.05 \times 10^{-27}$$

Or $$v = 1.05 \times 10^{-27} / 1.6 \times 10^{-27}$$

Or $$v = 0.63 \text{ m / s}$$

The required speed for hydrogen atom is 0.63 m / s.

11.5. The energy flux of sunlight reaching the surface of the earth is1.388 × 103 W/m2. How many photons (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.

Sol.

Given:
Energy flux = 1388 W / sq. m
wavelength = 550 nm

Energy of photon = hc / λ
= 3.61×10^{-19} J
So no. of photons = P / E
= 4×10^{21} photons / $m^2 s$

11.6. In an experiment on photoelectric effect, the slope of the cut-offvoltage versus frequency of incident light is found to be 4.12 × 10–15 V s. Calculate the value of Planck's constant.

Sol.

Slope = 4.12×10^{-15}
Or h / e = 4.12×10^{-15}
Putting value of e and solving

$h = 6.592 \times 10^{-34}$ J s

11.7. A 100W sodium lamp radiates energy uniformly in all directions.The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associatedwith the sodium light? (b) At what rate are the photons delivered tothe sphere?

Sol.

Given:

P = 100 W

λ = 589 nm

a) Energy per photon = hc / λ

$= 3.37 \times 10^{-19}$ J

b) No. of photons per second = P / E

$= 3 \times 10^{20}$ photons / second

11.8. The threshold frequency for a certain metal is 3.3 × 1014 Hz. If light of frequency 8.2 × 1014 Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission.

Sol.

Given:

$v_o = 3.3 \times 10^{14}$ Hz

$v = 8.2 \times 10^{14}$ Hz

Cut-off voltage

$eV_o = h(\nu - \nu_o)$

Substituting the values

$V_o = 2.03$ V

11.9. The work function for a certain metal is 4.2 eV. Will this metal givephotoelectric emission for incident radiation of wavelength 330 nm?

Sol.

$\phi_o = 4.2$ eV

Or $\nu_o h = 4.2$

Or $\nu_o = 1.01 \times 10^{15}$ Hz

Incident wavelength = 330 nm

Or $\nu = 6 \times 10^{-19}$ Hz

Since ν is less than ν_o, there will be no photoelectric emission.

11.10. Light of frequency 7.21 × 1014 Hz is incident on a metal surface.Electrons with a maximum speed of 6.0 × 105 m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

Sol.

Given:

$\nu = 7.21 \times 10^{14}$ Hz

$v = 6 \times 10^5$ m / s

Kinetic energy = h($\nu - \nu_o$)

Or $mv^2 / 2 = h(\nu - \nu_o)$

Solving the above equation

$\nu_o = 4.73 \times 10^{14}$ Hz

11.11. Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

Sol.

Given:

$\lambda = 488$ nm

Vo = 0.38 V

We know

eVo = hν – Φo

Or

Φo = hc / λ – eVo

= 3.46×10^{-19} J

= **2.16 eV**

11.12. Calculate the

a) momentum, and

b) de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

a) Using the relation

$$p = \sqrt{2meV}$$
$$= 4.04 \times 10^{-24} \text{ kg m / s}$$

b) And,

$$\lambda = h / p$$
$$= 0.164 \text{ nm}$$

11.13. What is the

a) momentum,

b) speed, and

c) de Broglie wavelength of an electron with kinetic energy of 120 eV.

Sol.

a) $p^2 = 2mK$

Or p = 5.91 x 10-24 kg m / s

b) Speed

$$mv^2 / 2 = K$$
$$mv^2 / 2 = 1.92 \times 10^{-17}$$
$$v = 6.5 \times 10^6 \text{ m / s}$$

c) $\lambda = h / p = 0.112$ nm

11.14. The wavelength of light from the spectral emission line of sodium is589 nm. Find the kinetic energy at which

a) an electron, and

b) a neutron, would have the same de Broglie wavelength.

Sol.

Given:

$\lambda = 589$ nm

For this λ,

$p = h / \lambda$

$= 1.12 \times 10^{-17}$ kg m / s

a) For electron,

$K = p^2 / 2m_e$

$= 6.9 \times 10^{-25}$ J

b) For neutron,

$K = p^2 / 2m_n$

$= 3.7 \times 10^{-28}$ J

11.15. What is the de Broglie wavelength of

a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s,

b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and

c) a dust particle of mass 1.0 × 10–9 kg drifting with a speed of 2.2 m/s?

Sol.

a) m = 0.04 kg

v = 1 km / s

$\lambda = h / mv$

$= 1.66 \times 10^{-35}$ m

b) m = 0.06 kg

v = 1 m / s

$\lambda = 1.1 \times 10^{-32}$ m

c) $m = 1 \times 10^{-9}$ kg

$v = 2.2$ m / s

$\lambda = 3 \times 10^{-25}$ m

11.16. An electron and a photon each have a wavelength of 1.00 nm. Find

a) Their momenta,

b) the energy of the photon, and

c) the kinetic energy of electron.

Sol.

a) Momentum of electron

$p = h / \lambda$

$= 6.63 \times 10^{-25}$ kg m / s

Momentum of photon

$p = h / \lambda$

$= 6.63 \times 10^{-25}$ kg m / s

b) Energy of photon

$E = mc^2$ ……………………………………(1)

Now,

$\lambda = h / mc$

Or $m = h / c\lambda$

Substituting value of m in eq. (1)

$E = hc / \lambda$

$= 1.24$ keV

c) Kinetic energy of electron

$K = p^2 / 2m$

$= 2.41 \times 10^{-19}$ J

= 1.51 eV

11.17.

a) For what kinetic energy of a neutron will the associated de Brogliewavelength be 1.40 × 10–10m?

b) Also find the de Broglie wavelength of a neutron, in thermalequilibrium with matter, having an average kinetic energy of(3/2) k T at 300 K.

Sol.

Given:

$\lambda = 1.4 \times 10^{-10}$ m

a) Now

$K = p^2 / 2m$

And $p = h / \lambda$

Therefore

$K = h^2 / 2m\lambda^2$

Or $K = 6.686 \times 10^{-21}$ J

b) Kinetic Energy

$K = (3 / 2)\, kT$

where

k = Boltzmann constant = 1.381×10^{-23} J / K

T = 300 K

So

$K = 6.21 \times 10^{-21}$ J

Now

$$\lambda = \frac{h}{\sqrt{2mK}} = 0.145 \text{ nm}$$

11.18. Show that the wavelength of electromagnetic radiation is equal tothe de Broglie wavelength of its quantum (photon).

Sol.

For a photon,
$p = h\nu / c$
Therefore,
$h / p = c / \nu = \lambda$
That is, the de Broglie wavelength of a photon equalsthe wavelength of electromagnetic radiation of which the photon is aquantum of energy and momentum.

11.19. What is the de Broglie wavelength of a nitrogen molecule in air at300 K? Assume that the molecule is moving with the root-meansquarespeed of molecules at this temperature. (Atomic mass ofnitrogen = 14.0076 u)

Sol.

Given:
T = 300 K
$m = 14.0076 \times 1.661 \times 10^{-27}$ kg
v = rms speed of molecules at T

K = 1.5 kT
Substitution results
$K = 6.21 \times 10^{-21}$ J

Now

$$\lambda = \frac{h}{\sqrt{2mK}}$$

which on solution gives

$\lambda = 0.038$ nm