

Chapter.9

Ray Optics and Optical Instruments

Class – XII

Subject – Physics

- 9.1.** A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

Sol.

Given:

$$u = -27 \text{ cm}$$

$$f = -18 \text{ cm}$$

$$h = 2.5$$

From the formula

$$1/v + 1/u = 1/f$$

$$1/v - 1/27 = -1/18$$

$$\text{Gives } v = -54 \text{ cm}$$

The image is real, inverted and magnified.

Now

$$-v/u = h'/h$$

Substitution yields

$$h' = 5 \text{ cm}$$

As $u \rightarrow f$, $v \rightarrow \infty$;

for $u < f$, image is virtual.

- 9.2.** A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

Sol.

Given:

$$h = 4.5 \text{ cm}$$

$$u = -12 \text{ cm}$$

$$f = 15 \text{ cm}$$

Putting the values in the formula used in previous question

$$v = 6.7 \text{ cm}$$

Magnification

$$m = -v / u$$

$$= 6.7 / 12$$

$$= 5 / 9$$

Therefore

$$h' = (5 / 9) \times 4.5$$

$$= 2.5 \text{ cm}$$

$$\text{As } u \rightarrow \infty, v \rightarrow f, m \rightarrow 0$$

- 9.3.** A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

Sol.

Given:

$$u = 12.5 \text{ cm}$$

$$v = 9.4 \text{ cm}$$

We know

$$\begin{aligned}\text{Refractive index of the medium} \\ &= \text{Real depth} / \text{Apparent depth} \\ &= 12.5 / 9.4 = \mathbf{1.33}\end{aligned}$$

Now

$$\begin{aligned}h' &= \text{real depth} / \text{refractive index of medium} \\ &= 12.5 / 1.63 \\ &= \mathbf{7.67 \text{ cm}}\end{aligned}$$

Microscope is situated at 9.4 cm

$$\begin{aligned}\text{So adjustment shift} &= 9.4 - 7.67 \\ &= \mathbf{1.7 \text{ cm}}\end{aligned}$$

- 9.4.** Figures 9.34(a) and (b) show refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface [Fig. 9.34(c)].

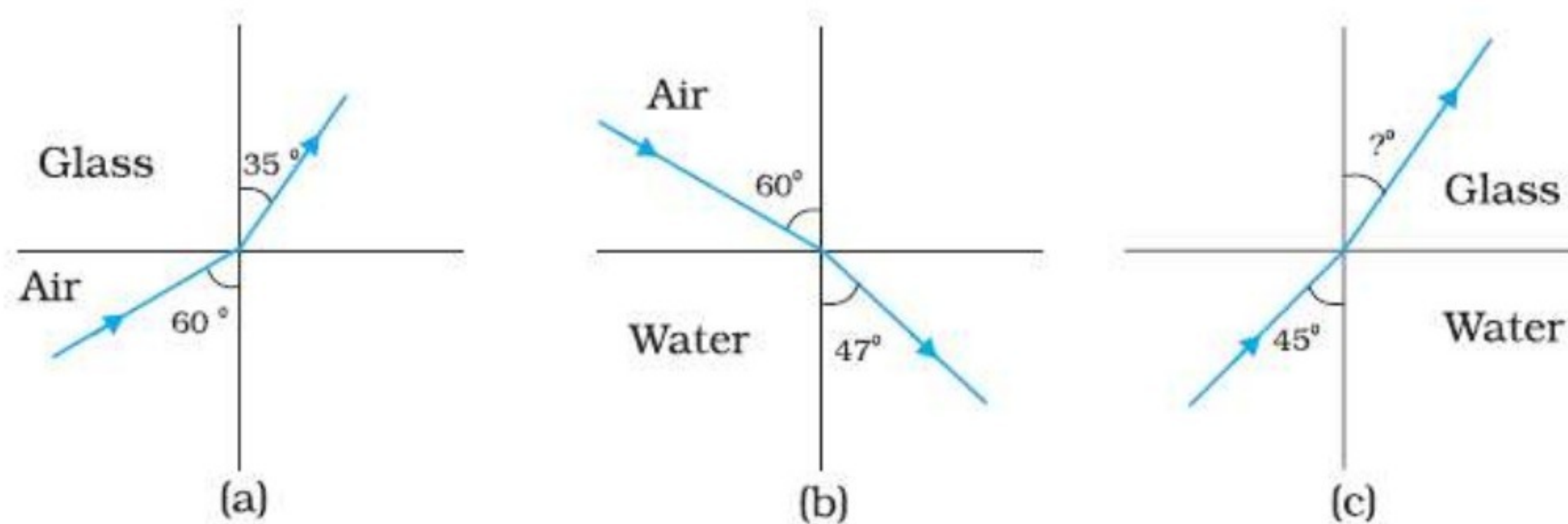


FIGURE 9.34

Sol.

For air-glass interface

$$\begin{aligned} n(\text{ga}) &= \sin i / \sin r \\ &= \sin 60 / \sin 35 \\ &= 1.51 \end{aligned}$$

For air-water interface

$$\begin{aligned} n(\text{wa}) &= \sin 60 / \sin 47 \\ &= 1.18 \end{aligned}$$

For glass-water interface

$$\begin{aligned} n(\text{gw}) &= n(\text{ga}) \times n(\text{aw}) \\ &= 1.51 \times (1 / 1.18) \\ &= 1.28 \end{aligned}$$

$$n(\text{gw}) = \sin 45 / \sin r$$

which gives

$$r = 33.54^\circ$$

- 9.5.** A small bulb is placed at the bottom of a tank containing water to a depth of 80cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)

Sol.

Given:

$$h = 80 \text{ cm}$$

$$n = 1.33$$

By the relation

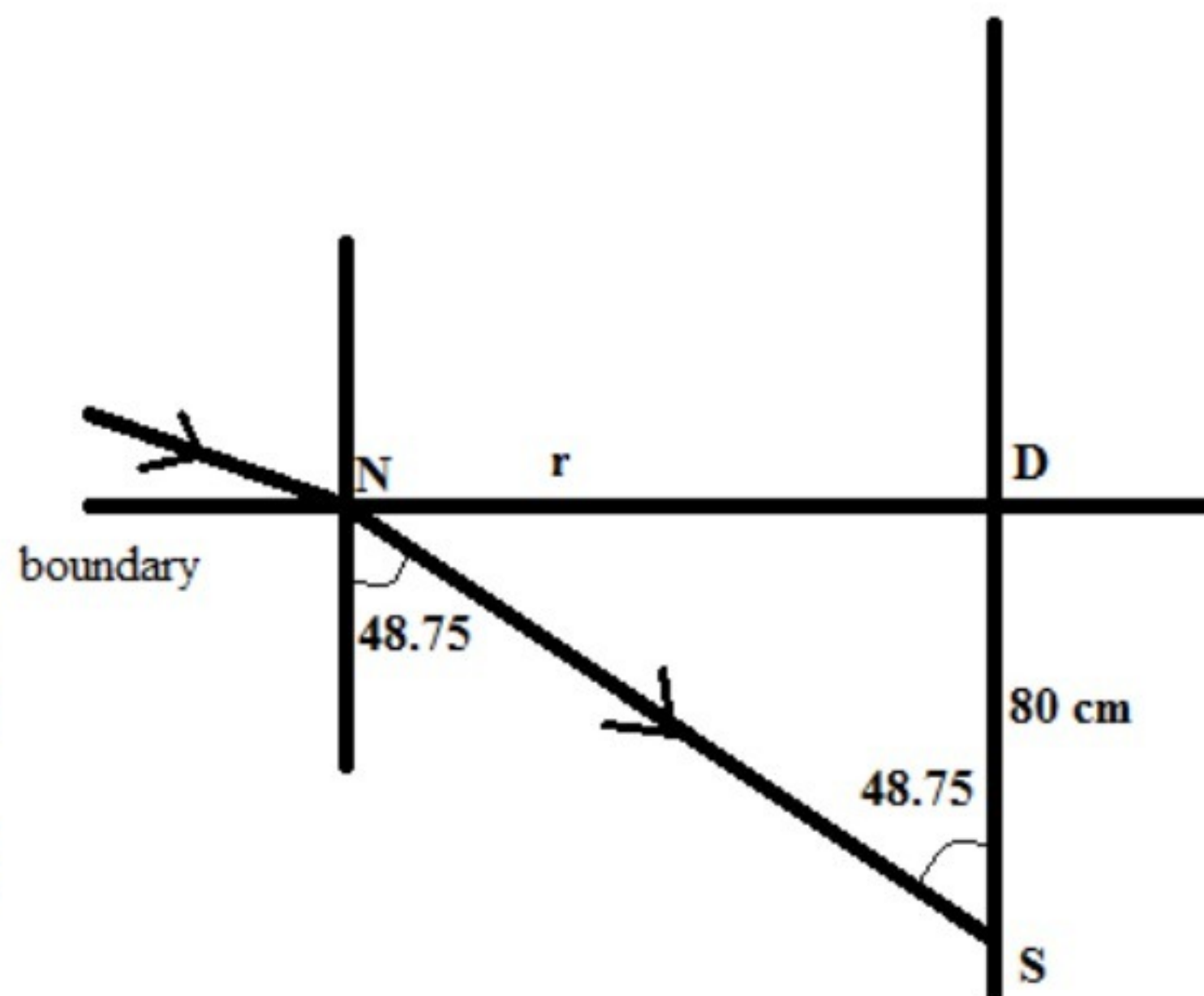
$$\sin i_c = 1 / n$$

$$i_c = 48.75$$

where i_c = critical angle

For rays to come out,

$$i < 48.75$$



In triangle NSD

$$\tan i_c = ND / 0.8$$

$$\text{Or } r = ND = 0.8 \times \tan i_c \\ = 0.912 \text{ m}$$

Thus

$$\text{Area} = \pi r^2$$

$$= 2.6 \text{ sq m}$$

- 9.6.** A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40° . What is the refractive index of the material of the prism? The refracting angle of the prism is 60° . If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.

Sol.

Given:

$$D_m = 40$$

$$A = 60$$

$$n(w) = 1.33$$

Refractive index of material of prism

$$n(p) = [\sin \{(A + D_m) / 2\}] / \sin(A / 2)$$

Substitution yields

$$n(p) = 1.53$$

Now in water

$$n(p) / n(w) = [\sin \{(A + D_m') / 2\}] / \sin(A / 2)$$

Or

$$(1.53 / 1.33) \cdot \sin 30 = \sin [\{(A + D_m') / 2\}]$$

Or

$$D_m' = 10^\circ$$

- 9.7.** Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature.

What is the radius of curvature required if the focal length is to be 20cm?

Sol.

Given:

$$n = 1.55$$

$$f = 20 \text{ cm}$$

Using Lens Maker's formula

$$n_1 / f = (n_2 - n_1) \cdot [1 / R + 1 / R]$$

Substituting values

$$1 / 20 = (1.55 - 1) \cdot [2 / R]$$

$$R = 22 \text{ cm}$$

9.8. A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12cm from P. At what point does the beam converge if the lens is

- a) a convex lens of focal length 20cm, and**
- b) a concave lens of focal length 16cm?**

Sol.

a) Convex $f = 20 \text{ cm}$

$$u = 12 \text{ cm}$$

$$1 / v - 1 / 12 = 1 / 20$$

Or

$$v = 7.5 \text{ cm}$$

b) Concave $f = 16 \text{ cm}$

$$u = 12 \text{ cm}$$

$$1/v - 1/12 = -1/16$$

Or

$$v = 48 \text{ cm}$$

9.9. An object of size 3.0cm is placed 14cm in front of a concave lens of focal length 21cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

Sol.

Given:

$$O = 3 \text{ cm}$$

$$u = -14 \text{ cm}$$

$$f = -21 \text{ cm}$$

Putting in formula

$$1/v + 1/14 = -1/21$$

Or

$$v = 8.4 \text{ cm}$$

Image is erect and virtual.

$$\text{As } u \rightarrow \infty, v \rightarrow f, m \rightarrow 0$$

9.10. What is the focal length of a convex lens of focal length 30cm in contact with a concave lens of focal length 20cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.

Sol.

Given:

$$f' = 30 \text{ cm (convex)}$$

$$f'' = -20 \text{ cm (concave)}$$

Effective focal length

$$1/f = 1/f' + 1/f''$$

which gives

$$f = -60 \text{ cm}$$

Negative sign points a diverging lens of focal length $f = 60 \text{ cm}$.

- 9.11.** A compound microscope consists of an objective lens of focal length 2.0cm and an eyepiece of focal length 6.25cm separated by a distance of 15cm. How far from the objective should an object be placed in order to obtain the final image at
- the least distance of distinct vision (25cm), and
 - at infinity? What is the magnifying power of the microscope in each case?

Sol.

Given:

$$f_o = 2 \text{ cm}$$

$$f_e = 6.25 \text{ cm}$$

$$L = 15 \text{ cm}$$

a) $D = 25 \text{ cm}$
 $v_e = 25 \text{ cm}$
 $f_e = 6.25 \text{ cm}$

By the formula

$$1 / v_e - 1 / u_e = 1 / f$$

Putting the values

$$- 1 / 25 - 1 / u = 1 / 6.25$$

Or

$$u = -5 \text{ cm}$$

Now,

$$v_o = 15 - 5$$

$$= 10 \text{ cm}$$

$$f_o = 2 \text{ cm}$$

Therefore

$$1 / 10 - 1 / u_o = 1 / 2$$

Or

$$u_o = -2.5 \text{ cm}$$

Magnifying power

$$m = \frac{10}{2.5} \cdot \left[1 + \frac{25}{6.25} \right]$$

Or

$$m = 20$$

b) Now

$$v_e = \infty$$

$$f_e = 6.25 \text{ cm}$$

Putting in formula

$$1 / \infty - 1 / u_e = 1 / 6.25$$

Or

$$u_e = -6.25 \text{ cm}$$

$$\begin{aligned}\text{Image distance for the objective} \\ &= 15 - 6.25 \\ &= 8.75 \text{ cm}\end{aligned}$$

Using lens formula
$$\frac{1}{8.75} - \frac{1}{u_o} = \frac{1}{2}$$

this gives
$$u_o = -2.59 \text{ cm}$$

Magnifying power
$$m = (8.75 / 2.59) \cdot (25 / 6.25)$$

$$= 13.5$$

9.12. A person with a normal near point (25cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5cm can bring an object placed at 9.0mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope.

Sol.

Given:

$$D = 25 \text{ cm}$$

$$f_o = 8 \text{ mm}$$

$$f_e = 2.5 \text{ cm}$$

$$u_o = 9 \text{ mm}$$

For image at 25 cm

Angular magnification of the eye-piece

$$\begin{aligned} &= (25 / 2.5) + 1 \\ &= 11 \end{aligned}$$

Using lens formula

$$-1 / 25 - 1 / u_e = 1 / 2.5$$

Or

$$u_e = -2.27 \text{ cm}$$

Also,

$$1 / v_o - (-1 / 0.9) = 1 / 0.8$$

Or

$$v_o = 7.2 \text{ cm}$$

$$\begin{aligned} \text{Therefore, Separation} &= 2.27 + 7.2 \\ &= \mathbf{9.47 \text{ cm}} \end{aligned}$$

Magnifying power

$$\begin{aligned} &= (7.2 / 0.9) \cdot [1 + \{25 / 2.5\}] \\ &= \mathbf{88} \end{aligned}$$

- 9.13.** A small telescope has an objective lens of focal length 144cm and an eyepiece of focal length 6.0cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

Sol.

Given:

$$f_o = 144 \text{ cm}$$

$$f_e = 6 \text{ cm}$$

Length of the telescope tube

$$\begin{aligned} &= f_o + f_e \\ &= 150 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Magnification} &= f_o / f_e \\ &= 24 \end{aligned}$$

9.14.

a) A giant refracting telescope at an observatory has an objective lens of focal length 15m. If an eyepiece of focal length 1.0cm is used, what is the angular magnification of the telescope?

b)

If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is $3.48 \times 10^6 \text{m}$, and the radius of lunar orbit is $3.8 \times 10^8 \text{m}$.

Sol.

a) Given:

$$f_o = 15 \text{ m}$$

$$f_e = 1 \text{ cm}$$

Angular magnification

$$= f_o / f_e$$

$$= 1500$$

b) Dia = $3.48 \times 10^6 \text{ m}$

$$\text{Rad} = 3.8 \times 10^8 \text{ m}$$

Then, angles subtended

$$3.48 \times 10^6 / 3.8 \times 10^8 = D / 15$$

Or

$$D = 13.7 \text{ cm}$$

9.15. Use the mirror equation to deduce that:

- a) an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.
- b) a convex mirror always produces a virtual image independent of the location of the object.
- c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.
- d) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

[Note: This exercise helps you deduce algebraically properties of images that one obtains from explicit ray diagrams.]

Sol.

a) $1/v + 1/u = 1/f$

$$f < 0$$

$$u < 0$$

$$\text{So } 1/v - 1/u = -1/f$$

$$\text{At } u = f$$

$$1/v - 1/f = -1/f$$

Or

$$v = \infty$$

$$\text{At } u = 2f$$

$$1/v - 1/2f = -1/f$$

Or

$$v = -2f$$

$$\text{At } u = 1.5f$$

$$1/v - 1/1.5f = -1/f$$

Or

$$v = 2f$$

Hence deduced.

b) $f > 0$

$$u < 0$$

$$\text{At } u = \infty$$

$$1/v = 1/f$$

Or

$$v = f \quad (\text{+ive})$$

$$\text{At } u = f$$

$$1/v - 1/f = 1/f$$

Or

$$v = f/2 \quad (\text{+ive})$$

Hence always virtual image is formed.

c) $f > 0$

$$u < 0$$

Therefore

$$1/v = 1/f - 1/u$$

Since $u < 0$

$$1/v > 1/f$$

Or
 $v < f$
Hence image is diminished.

d) $f < 0$
 $f < u < 0$

$$1/v - 1/u = -1/f$$

Or
 $1/v = [-1/f + 1/u] < 0$
So virtual image.

As $m = v/u$
 $u < 0$
 $v > 0$
 $v > u$
So
 $v/u > 1$
Image is enlarged.

9.16. A small pin fixed on a table top is viewed from above from a distance of 50cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

Sol.

Given:
 $n = 1.5$
Actual depth, $d = 15$ cm
Apparent depth = d'

We know

$$n = d / d'$$

$$1.5 = 15 / d'$$

$$d' = 10 \text{ cm}$$

$$\begin{aligned}\text{So the distance} &= d - d' \\ &= 5 \text{ cm}\end{aligned}$$

9.17.

- a) Figure 9.35 shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure.
- b) What is the answer if there is no outer covering of the pipe?

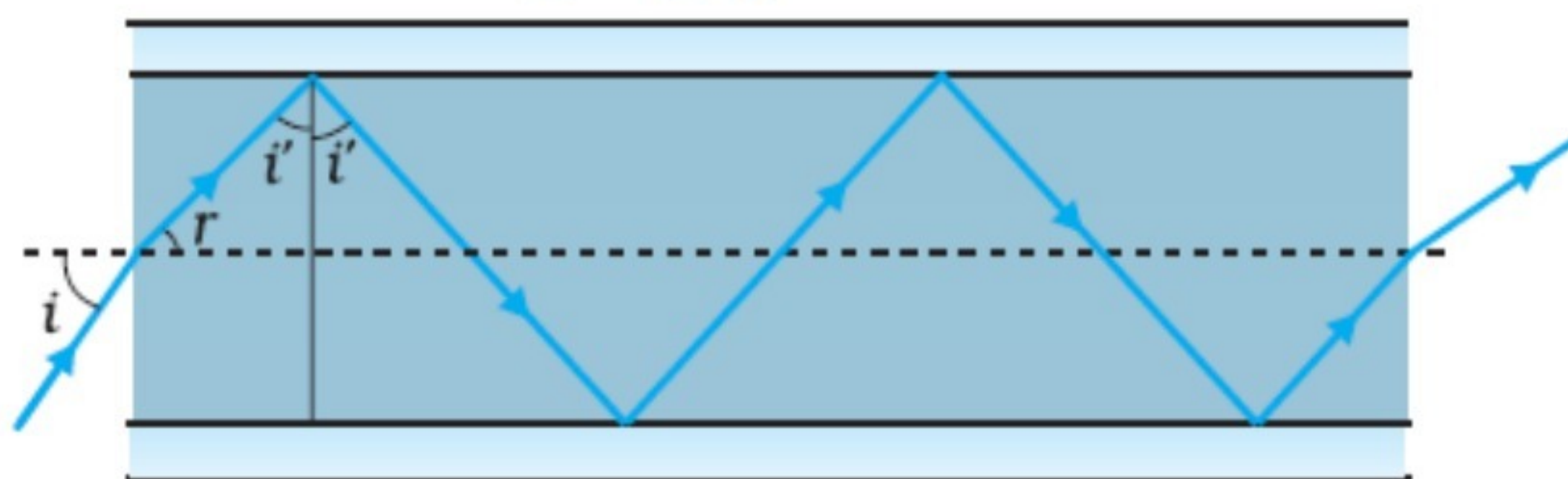


FIGURE 9.35

- a) Given:
- $$n_g = 1.68$$
- $$n_p = 1.44$$

By Snell's law

$$\sin i_c = 1.44 / 1.68$$
$$i_c = 59$$

Thus range is

$$0 < i < 59$$

b) No outer covering

$$\sin i_c' = 1 / 1.68$$

$$i_c' = 36.5$$

Thus range

$$53.5 < i < 90$$

9.18. Answer the following questions:

- You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real images under some circumstances? Explain.
- A virtual image, we always say, cannot be caught on a screen. Yet when we 'see' a virtual image, we are obviously bringing it on to the 'screen' (i.e., the retina) of our eye. Is there a contradiction?
- A diver under water, looks obliquely at a fisherman standing on the bank of a lake. Would the fisherman look taller or shorter to the diver than what he actually is?
- Does the apparent depth of a tank of water change if viewed obliquely? If so, does the apparent depth increase or decrease?
- The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to a diamond cutter?

Sol.

- A plane or convex mirror can produce a real image if the object is virtual.

- b) There is no contradiction as the screen here is not located at the position of the virtual image.
- c) Taller.
- d) The apperent depth decreases.
- e) The diamond can be cut in a way so that light goes total internal reflection producing sparkling effect

9.19. The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?

Sol.

$$\begin{aligned}\text{Maximum possible focal length of the lens} \\ &= \text{distance} / 4 \\ &= 3 / 4 \\ &= \mathbf{0.75 \text{ m}}\end{aligned}$$

9.20. A screen is placed 90cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20cm. Determine the focal length of the lens.

Sol.

Given:

$$D = 90 \text{ cm}$$

$$d = 20 \text{ cm}$$

By the formula

$$f = (D^2 - d^2) / 4D$$

Substitution yields

$$f = 21.4 \text{ cm}$$

9.21.

- a) Determine the ‘effective focal length’ of the combination of the two lenses in Exercise 9.10, if they are placed 8.0cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all?
- b) An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40cm. Determine the magnification produced by the two-lens system, and the size of the image.

Sol.

- a) (i) Concave lens on left side

$$f_1 = -20 \text{ cm}$$

$$u_1 = -\infty$$

Therefore

$$1/v_1 + 1/\infty = -1/20$$

$$v_1 = -20 \text{ cm}$$

This serves as object for second lens

$$f_2 = 30 \text{ cm}$$

$$u_2 = -28 \text{ cm}$$

Therefore

$$\frac{1}{v_2} + \frac{1}{28} = \frac{1}{30}$$
$$v_2 = -420 \text{ cm}$$

(ii) Convex lens on left

$$f_1 = 30 \text{ cm}$$

$$u_1 = -\infty$$

Putting in formula

$$\frac{1}{v_1} + \frac{1}{\infty} = \frac{1}{30}$$

$$v_1 = 30 \text{ cm}$$

This serves as object for second lens

$$f_2 = -20 \text{ cm}$$

$$u_2 = 22 \text{ cm}$$

Therefore

$$\frac{1}{v_2} - \frac{1}{22} = -\frac{1}{20}$$

$$v_2 = -220 \text{ cm}$$

The answer depends on which side of lens system the parallel beam is incident. There is no use of concept of effective focal length here.

b) Given:

$$u_1 = -40 \text{ cm}$$

$$f_1 = 30 \text{ cm}$$

Therefore

$$\frac{1}{v_1} + \frac{1}{40} = \frac{1}{30}$$

Or

$$v_1 = 120 \text{ cm}$$

Magnification due to convex lens = 3

$$u_2 = 112 \text{ cm}$$

$$f_2 = -20 \text{ cm}$$

$$1/v_2 - 1/112 = -1/20$$

Or

$$v_2 = -24.3 \text{ cm}$$

$$\text{Magnification due to concave lens} = 20/92$$

$$\text{Net magnitude} = 0.652$$

$$\text{Size of image} = 0.98 \text{ cm}$$

9.22. At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

Sol.

Given:

$$n = 1.524$$

$$r = 60^\circ$$

$$\sin i_c = 1/1.524$$

$$i_c = 41^\circ$$

$$\text{Thus } r = 19^\circ$$

$$\sin i = 0.4962$$

$$\text{Or } i = 30^\circ$$

- 9.23.** You are given prisms made of crown glass and flint glass with a wide variety of angles. Suggest a combination of prisms which will
- deviate a pencil of white light without much dispersion,
 - disperse (and displace) a pencil of white light without much deviation.

Sol.

- First prism is taken of crown glass.
Second prism is taken of flint glass.
Refracting angle of second prism is so chosen such that it is less than that of crown glass prism, so that dispersion due to the first is nullified by the second.
- The angle of first glass prism is increased to disperse without deviation, so that deviations due to the two prisms are equal and opposite. First glass prism is taken of greater angle.

- 9.24.** For a normal eye, the far point is at infinity and the near point of distinct vision is about 25cm in front of the eye. The cornea of the eye provides a converging power of about 40 dioptres, and the least converging power of the eye-lens behind the cornea is about 20 dioptres. From this rough data estimate the range of accommodation (i.e., the range of converging power of the eye-lens) of a normal eye.

Sol.

Least converging power to see object at $\infty = 60$ D

So distance between retina and cornea eye lens = $5/3$ cm

At near point

$$u = -25 \text{ cm}$$

$$v = 5/3 \text{ cm}$$

$$f = 25/16 \text{ cm}$$

Corresponding power = 64 D

Therefore

$$\text{Power of the eye lens} = 64 - 40 = 24 \text{ D}$$

Range of accommodation of the eye lens = 20 to 24 dioptres

9.25. Does short-sightedness (myopia) or long-sightedness (hypermetropia) imply necessarily that the eye has partially lost its ability of accommodation? If not, what might cause these defects of vision?

Sol.

No, a myopic or hypermetropic person may have normal ability of accommodation of the eye lens. The partial loss of ability of accommodation of eye lens is called presbyopia. It can be corrected in the way hypermetropia is dealt with.

9.26. A myopic person has been using spectacles of power -1.0 dioptre for distant vision. During old age he also needs to use separate reading glass of power $+ 2.0$ dioptres. Explain what may have happened.

Sol.

The person uses lens of power -1.0 dioptre because his far point is 100 cm, where a virtual image is formed by the lens. In old age, he uses a lens of $+ 2.0$ D since the normal near point of vision ceases to be 25 cm. The lens forms the virtual image of object during reading (or near point) at 50 cm. This defect is called presbyopia, which is common in old age due to loss of ability of accommodation of eyes.

- 9.27.** A person looking at a person wearing a shirt with a pattern comprising vertical and horizontal lines is able to see the vertical lines more distinctly than the horizontal ones. What is this defect due to? How is such a defect of vision corrected?

Sol.

When the curvature of retina is not same in every plane the defect is called astigmatism. The vertical lines can be seen by the person because the curvature in vertical plane is normal forming the images at retina. The images of horizontal lines are not formed because the curvature in horizontal plane is not sufficient. Such defect can be corrected by using cylindrical lens. In this case the cylindrical lens with vertical axis is required.

- 9.28.** A man with normal near point (25 cm) reads a book with small print using a magnifying glass: a thin convex lens of focal length 5 cm.
- What is the closest and the farthest distance at which he should keep the lens from the page so that he can read the book when viewing through the magnifying glass?
 - What is the maximum and the minimum angular magnification (magnifying power) possible using the above simple microscope?

Sol.

Given:

$$f = 5 \text{ cm}$$

- a) By the formula

$$1/v - 1/u = 1/f$$

Substituting the values

$$-1/25 - 1/u = 1/5$$

which gives

$$u = -4.2 \text{ cm}$$

This is the closest distance at which the person can read the book.

For the farthest distance

$$v' = \infty$$

Using the above formula

$$1 / \infty - 1 / u' = 1 / 5$$

which gives

$$u' = -5 \text{ cm}$$

This is the farthest distance.

b) Maximum angular magnification

$$\begin{aligned} m &= 25 / 4.167 \\ &= 6 \end{aligned}$$

Minimum angular magnification

$$m' = 25 / 5 = 5$$

9.29. A card sheet divided into squares each of size 1 mm² is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 9 cm) held close to the eye.

- What is the magnification produced by the lens? How much is the area of each square in the virtual image?
- What is the angular magnification (magnifying power) of the lens?
- Is the magnification in (a) equal to the magnifying power in (b)? Explain.

Sol.

a) Using the formula of previous question

$$1/v + 1/9 = 1/10$$

Or

$$v = -90 \text{ cm}$$

Magnitude of magnification

$$m = 90/9$$

$$= 10$$

Area of each square in virtual image

$$= 10 \times 10 \times 1$$

$$= 100 \text{ sq mm}$$

$$= 1 \text{ sq cm}$$

b) Angular magnification

$$= 25/9$$

$$= 2.8$$

c) No, both are two different things. Magnification magnitude is $|v/u|$, while magnifying power is $25/|u|$. These are equal only when the image is located at the near point of 25 cm.

9.30.

a) At what distance should the lens be held from the figure in Exercise 9.29 in order to view the squares distinctly with the maximum possible magnifying power?

b) What is the magnification in this case?

c) Is the magnification equal to the magnifying power in this case? Explain.

Sol.

- a) When the image is at near point of 25 cm, the magnification achieved is maximum.

So,

$$1 / v - 1 / u = 1 / f$$

Putting the values

$$-1 / 25 - 1 / u = 1 / 10$$

which yields

$$u = -7.14 \text{ cm}$$

- b) Magnification = v / u

$$= 25 / 7.14$$

$$= 3.5$$

- c) Yes, since the image is produced at 25 cm. At 25 cm, magnification magnitude is same as magnifying power.

9.31. What should be the distance between the object in Exercise 9.30 and the magnifying glass if the virtual image of each square in the figure is to have an area of 6.25 mm². Would you be able to see the squares distinctly with your eyes very close to the magnifier?

[Note: Exercises 9.29 to 9.31 will help you clearly understand the difference between magnification in absolute size and the angular magnification (or magnifying power) of an instrument.]

Sol.

Given:

Area of virtual image of each square, $A = 6.25 \text{ sq mm}$

Area of each image, $A' = 1 \text{ sq mm}$

Therefore

$$\begin{aligned}\text{Magnification} &= \sqrt{\frac{A}{A'}} \\ &= 2.5\end{aligned}$$

$$\text{As } v = 2.5 u$$

$$\text{So } 1 / 2.5u - 1 / u = 1 / 10$$

giving

$$u = -6 \text{ cm}$$

$$|v| = 15 \text{ cm}$$

Since v is less than the normal near point of 25 cm, virtual image cannot be seen by the eye distinctly.

9.32. Answer the following questions:

- The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?
- In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back?
- Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?
- Why must both the objective and the eyepiece of a compound microscope have short focal lengths?
- When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece?

Sol.

- a) Angular magnification can be achieved by placing object much closer than 25 cm. This is exactly what magnifying glasses do.
- b) Yes. This is so because the angle subtended at the eye is slightly less than the angle subtended at the lens.
- c) It is not easy to make lens of very small focal length. Moreover, the spherical and chromatic aberrations increases with decreasing focal length.
- d) For small f_e , angular magnification increases. It is given by $(25 / f_e) + 1$

$$\begin{aligned}\text{Magnification of the objective} \\ &= 1 / [l_{uo} / f_o] - 1\end{aligned}$$

For $l_{uo} > f_o$, m is large. So f_o is small.

- e) To maximize our field of view, the light refracted by objective should be collected. This is achieved at eye-ring, the correct location of which depends on the separation between the objective and the eye-piece.

9.33. An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25cm and an eyepiece of focal length 5cm. How will you set up the compound microscope?

Sol.

Given:

$$f_o = 1.25 \text{ cm}$$

$$f_e = 5 \text{ cm}$$

$$\begin{aligned}\text{Angular magnification of eye-piece at 25 cm} \\ &= (25 / 5) + 1 \\ &= 6\end{aligned}$$

Magnification of objective

$$= 30 / 6$$
$$= 5$$

Since $v_o / u_o = 5$

Therefore

$$1 / 5u_o - 1 / u_o = 1 / 1.25$$

which yields

$$u_o = -1.5 \text{ cm}$$

And $v_o = 7.5 \text{ cm}$

For eyepiece

$$|u_e| = 25 / 6$$

$$= 4.17 \text{ cm}$$

Total separation between objective and eye-piece

$$= 7.5 + 4.17$$

$$= 11.67 \text{ cm}$$

For the desired magnification the objective should be placed 1.5 cm away from the objective.

9.34. A small telescope has an objective lens of focal length 140cm and an eyepiece of focal length 5.0cm. What is the magnifying power of the telescope for viewing distant objects when

- the telescope is in normal adjustment (i.e., when the final image is at infinity)?
- the final image is formed at the least distance of distinct vision (25cm)?

Sol.

Given:

$$f_o = 140 \text{ cm}$$

$$f_e = 5 \text{ cm}$$

a) For normal adjustment

$$\begin{aligned}m &= f_o / f_e \\&= 140 / 5 \\&= 28\end{aligned}$$

b) For distinct vision

$$\begin{aligned}m &= [f_o / f_e].[1 + \{f_e / 25\}] \\ \text{Substitution yields} \\ \mathbf{m} &= \mathbf{33.6}\end{aligned}$$

9.35.

- a) For the telescope described in Exercise 9.34 (a), what is the separation between the objective lens and the eyepiece?
- b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?
- c) What is the height of the final image of the tower if it is formed at 25cm?

Sol.

a) The separation will be

$$\begin{aligned}&= f_o + f_e \\&= \mathbf{145 \text{ cm}}\end{aligned}$$

b) Angle subtended by the tower

$$\begin{aligned}&= h / u \\&= 100 / 3000 \\&= 1 / 30 \text{ rad}\end{aligned}$$

Angle subtended by the image produced by the objective

$$= h' / f_o$$

$$= h' / 140$$

And

$$h' / 140 = 1 / 30$$

$$\text{Or } h' = 4.7 \text{ cm}$$

c) Magnification of the eye-piece

$$= 1 + (d / f_e)$$

$$= 1 + (25 / 5)$$

$$= 6$$

Height of the final image

$$= m.h'$$

$$= 6 \times 4.7$$

$$= 28 \text{ cm}$$

9.36. A Cassegrain telescope uses two mirrors as shown in Fig. 9.33. Such a telescope is built with the mirrors 20mm apart. If the radius of curvature of the large mirror is 220mm and the small mirror is 140mm, where will the final image of an object at infinity be?

Sol.

Given:

$$d = 20 \text{ mm}$$

$$R_1 = 220 \text{ mm}$$

$$R_2 = 140 \text{ mm}$$

$$\text{So } f_1 = 220 / 2 = 110 \text{ mm}$$

$$\text{And } f_2 = 140 / 2 = 70 \text{ mm}$$

Large mirror forms an image, which is virtual object for smaller mirror.

So

$$u = 110 - 20 = 90 \text{ mm}$$

Using mirror formula

$$\frac{1}{v} + \frac{1}{90} = \frac{1}{70}$$

which produces

$$v = 315 \text{ mm}$$

- 9.37.** Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in Fig. 9.36. A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?

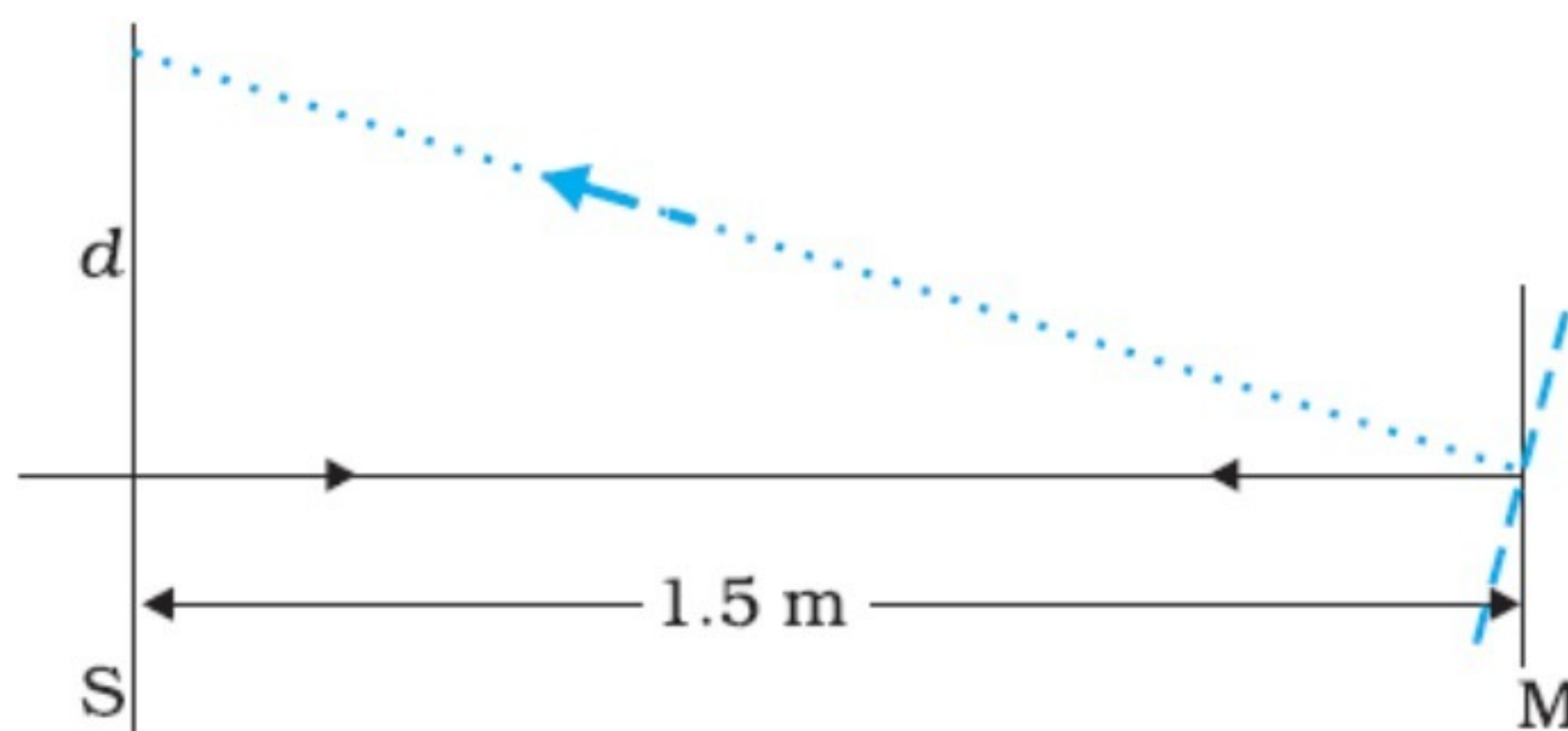


FIGURE 9.36

Sol.

The deflection of reflected rays is twice the angle of rotation of the mirror.

$$\text{So } 2A = 7^\circ$$

Thus

$$d = 1.5 \times \tan 2A$$

$$= 18.4 \text{ cm}$$

- 9.38.** Figure 9.37 shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane

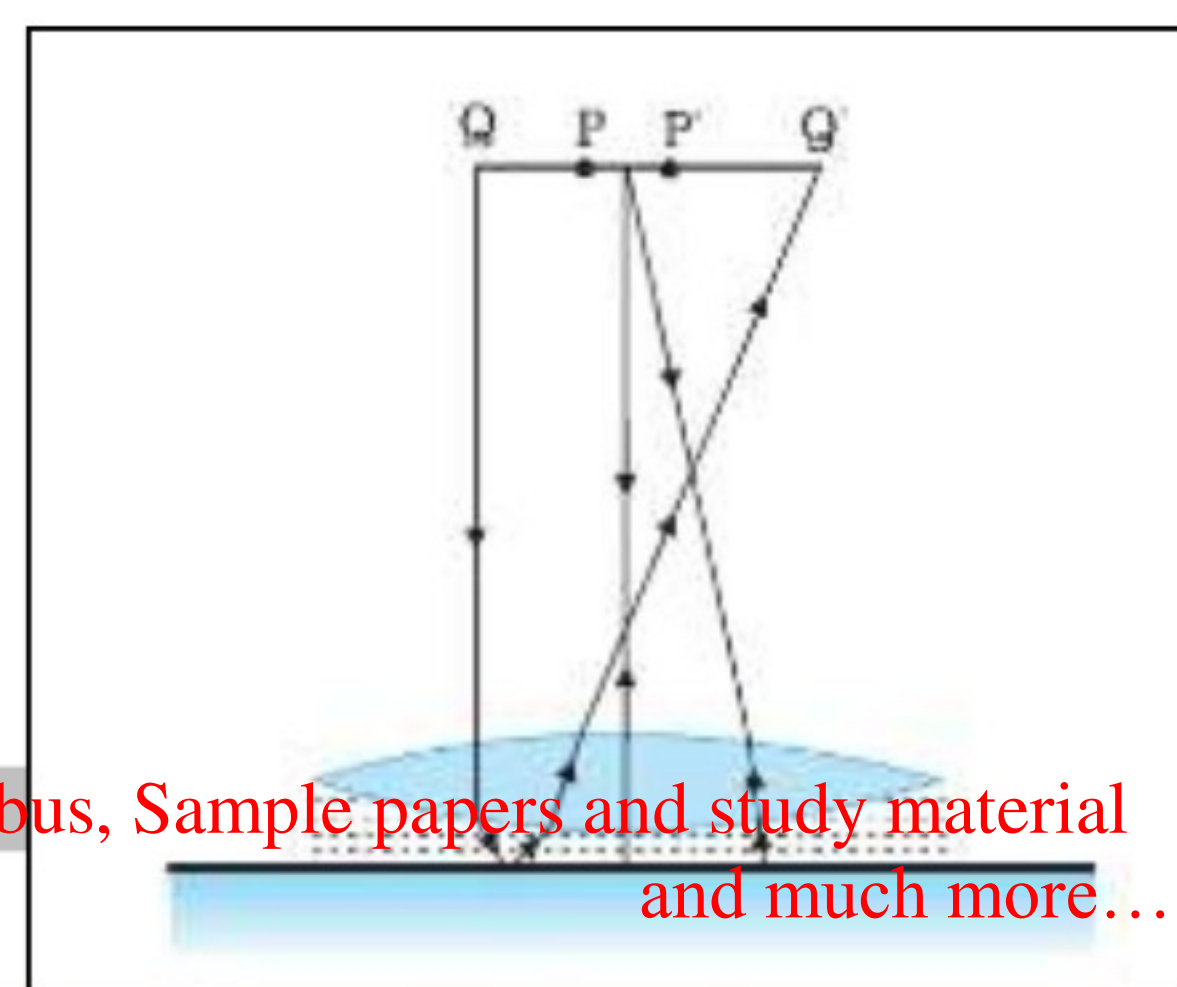


FIGURE 9.37

mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0cm. What is the refractive index of the liquid?

Sol.

Given:

$$n = 1.5$$

$$d = 45 \text{ cm}$$

$$d' = 30 \text{ cm}$$

Rearranging the given information

Focal length of equiconvex lens and liquid

$$f = 45 \text{ cm}$$

Focal length of the equiconvex lens

$$f' = 30 \text{ cm}$$

Therefore the equivalent focal length

$$1 / f = 1 / f' + 1 / f''$$

$$\text{Or } 1 / 45 = 1 / 30 + 1 / f''$$

$$\text{Or } f'' = -90 \text{ cm}$$

Now, by the relation

$$\frac{1}{f''} = (n - 1) \left[\frac{1}{R} + \frac{1}{R} \right]$$

Substituting the values

$$1 / 30 = (0.5) \cdot [2 / R]$$

Or $R = 30 \text{ cm}$

In case of liquid

$$1 / f' = (n' - 1) \cdot [1 / R - 1 / \infty]$$

where

n' = refractive index of the liquid

R' of plane mirror is infinite.

Solving the above expression

$$- 1 / 90 = (n' - 1) \cdot [(1 / 30) - 0]$$

Or $n' - 1 = 1 / 3$

which gives

$$n' = 4 / 3$$

$$= 1.33$$