Simplifying Test Prep

## Chapter. 8 <br> Electromagnetic Waves <br> Class - XII <br> Subject - Physics

8.1. Figure 8.6 shows a capacitor made of two circular plates each of radius 12 cm , and separated by 5.0 cm . The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15 A .

a) Calculate the capacitance and the rate of charge of potential difference between the plates.
b) Obtain the displacement current across the plates.
c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.

Sol.

Given:
$\mathrm{r}=0.12 \mathrm{~m}$
$\mathrm{d}=0.05 \mathrm{~m}$
$\mathrm{I}=0.15 \mathrm{~A}$
a) We know
$c=\epsilon_{b} \cdot \frac{A}{d}$

Calculating area
$\mathrm{A}=3.14 \times 0.12 \times 0.12$
$\mathrm{A}=0.045$ sq. m
Substituting in above eqn
$\mathrm{C}=80.1 \mathrm{pF}$ Now,
Rate of change of potential difference $\mathrm{dQ} / \mathrm{dt}=\mathrm{C} . \mathrm{dV} / \mathrm{dt}$
Or
$\mathrm{dV} / \mathrm{dt}=\mathrm{I} / \mathrm{C}$
Thus $\mathrm{dV} / \mathrm{dt}=0.15 / 80.1 \mathrm{p}$
Or $\quad \mathrm{dV} / \mathrm{dt}=1.87 \times 10^{9} \mathrm{~V} / \mathrm{s}$
b) Displacement current is given by
$\dot{i}_{d t}=\epsilon_{o} \frac{d \Phi_{E}}{d t}$
And $\mathrm{T}_{\ddot{\epsilon}}=\mathrm{E} . \mathrm{A}$
So $\quad d \Phi_{\ddot{E}} / \mathrm{dt}=\mathrm{A} . \mathrm{dE} / \mathrm{dt}$
Now
$\mathrm{E}=\mathrm{Q} / c_{c} . \mathrm{A}$
Differentiating it w.r.t. time
$\mathrm{dE} / \mathrm{dt}=\mathrm{i} / c_{c} . \mathrm{A}$
Substituting these simplified values in eq. (1)
$\mathrm{i}_{\mathrm{d}}=c_{\varepsilon} . \mathrm{A} . \mathrm{i} / c_{\varepsilon} . \mathrm{A}=\mathrm{i}$
Therefore
$\mathrm{i}_{\mathrm{d}}=\mathrm{i}=0.15 \mathrm{~A}$
c) It will be valid if both the conduction current and displacement current are taken into account.
8.2. A parallel plate capacitor (Fig. 8.7) made of circular plates each of radius $R=6.0 \mathrm{~cm}$ has a capacitance $C=100 \mathrm{pF}$. The capacitor is connected to a 230 V ac supply with a (angular) frequency of 300 rad s-1.

a) What is the rms value of the conduction current?
b) Is the conduction current equal to the displacement current?
c) Determine the amplitude of $B$ at a point 3.0 cm from the axis between the plates.

Given:
$\mathrm{R}=0.06 \mathrm{~m}$
$\mathrm{C}=100 \mathrm{pF}$
$\mathrm{V}=230 \mathrm{~V}$
$\mathrm{w}=300 \mathrm{rad} / \mathrm{s}$
a) $\operatorname{Irms}=\mathrm{V} . \mathrm{wC}$

Substitution yields
Irms $=6.9$ micro-amperes
b) Yes
c) For oscillating B and i

$$
\begin{aligned}
& B_{o}=\frac{\mu_{o}}{2 \pi} \cdot \frac{r}{R^{2}} \cdot \dot{i}_{o} \\
& \mathrm{i}_{\mathrm{o}}=1.414 \times \mathrm{Irms}
\end{aligned}
$$

Putting the required values

$$
\mathrm{B}_{\mathrm{o}}=1.63 \times 10^{-11} \mathrm{~T}
$$

8.3. What physical quantity is the same for $X$-rays of wavelength $\mathbf{1 0 - 1 0}$ m , red light of wavelength $6800 \AA$ and radio waves of wavelength 500 m ?

Sol.

Speed c. All electromagnetic waves travel with same speed.
8.4. A plane electromagnetic wave travels in vacuum along $z$-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz , what is its wavelength?

Sol.

E and B will be mutually perpendicular and perpendicular to direction of propagation in $\mathrm{x}-\mathrm{y}$ plane.
Wavelength $=\mathrm{c} / \mathrm{f}$

$$
\begin{aligned}
& =3 \times 10^{8} / 30 \times 10^{6} \\
& =10 \mathrm{~m}
\end{aligned}
$$

8.5. A radio can tune in to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band? Sol.
$\lambda_{1}=c / f_{l}$
$=40 \mathrm{~m}$
$\lambda_{2}=c / f_{2}$
$=25 \mathrm{~m}$
Wavelength band $=40 \mathrm{~m}-25 \mathrm{~m}$
8.6. A charged particle oscillates about its mean equilibrium position with a frequency of 109 Hz . What is the frequency of the electromagnetic waves produced by the oscillator?

Sol. Same, i.e., $10^{9} \mathrm{~Hz}$.
8.7. The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is $B 0=510 \mathrm{nT}$. What is the amplitude of the electric field part of the wave?

Sol.

Given $\mathrm{Bo}=510 \mathrm{nT}$
Using the relation
Eo / Bo = c
Eo $=\mathrm{c}$. Bo
Eo $=\left(3 \times 10^{8}\right) \cdot\left(510 \times 10^{-9}\right)$
Eo $=153 \mathrm{~N} / \mathrm{C}$
8.8. Suppose that the electric field amplitude of an electromagnetic wave is $E 0=120 \mathrm{~N} / \mathrm{C}$ and that its frequency is $v=50.0 \mathrm{MHz}$.
a) Determine, $\mathrm{B} 0, \omega, \mathrm{k}$, and $\lambda$.
b) Find expressions for $\mathbf{E}$ and $\mathbf{B}$.

Sol.

Given:
Eo $=120 \mathrm{~N} / \mathrm{C}$
$v=50 \mathrm{MHz}$
a) $\mathrm{Bo}=\mathrm{Eo} / \mathrm{c}$

Or $\mathrm{Bo}=400 \mathrm{nT}$

$$
\lambda=c / v
$$

Or $\lambda=6 m$
$\mathrm{k}=2 \pi / \lambda$
Or $\mathrm{k}=1.05 \mathrm{rad} / \mathrm{m}$

$$
w=\mathrm{c} . \mathrm{k}
$$

Or $w=3.14 \times 10^{8} \mathrm{rad} / \mathrm{s}$
b) $\mathrm{E}=\mathrm{E}_{\mathrm{o}} \sin (\mathrm{kz}-\mathrm{wt})$
$\mathrm{E}=120 \sin \left[1.05 \mathrm{x}-\left(3.14 \times 10^{8}\right) \mathrm{t}\right]$

$$
\begin{aligned}
& \mathrm{B}=\mathrm{B}_{\mathrm{o}} \sin (\mathrm{kx}-\mathrm{wt}) \\
& \mathrm{B}=400 \times 10^{-9} \sin \left[1.05 \mathrm{x}-\left(3.14 \times 10^{8}\right) \mathrm{t}\right] \tilde{k}
\end{aligned}
$$

8.9. The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula $E=h v$ (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for
different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

Sol.

We know
$\mathrm{E}=\mathrm{h} v=\mathrm{hc} / \lambda$
Solving for different parts of electromagnetic spectrum
a) Long Radio Waves

$$
\begin{aligned}
& \lambda=10^{5} \mathrm{~m} \\
& \mathrm{E}=1.24 \times 10^{-11} \mathrm{eV}=12.4 \mathrm{peV}
\end{aligned}
$$

b) AM Radio

$$
\begin{aligned}
& \lambda=100 \mathrm{~m} \\
& \mathrm{E}=1.24 \times 10^{-8} \mathrm{eV}=12.4 \mathrm{neV}
\end{aligned}
$$

c) Television and FM Radio

$$
\begin{aligned}
& \lambda=10 \mathrm{~m} \\
& \mathrm{E}=1.24 \times 10^{-7} \mathrm{eV}=0.124 \mu \mathrm{eV}
\end{aligned}
$$

d) Short Radio Waves

$$
\begin{aligned}
& \lambda=10^{-1} \mathrm{~m} \\
& \mathrm{E}=1.24 \times 10^{-5} \mathrm{eV}=12.4 \mu \mathrm{eV}
\end{aligned}
$$

e) Microwaves

$$
\begin{aligned}
& \lambda=10^{-2} \mathrm{~m} \\
& \mathrm{E}=1.24 \times 10^{-4} \mathrm{eV}=0.124 \mathrm{meV}
\end{aligned}
$$

f) Infrared

$$
\begin{aligned}
& \lambda=10^{-4} \mathrm{~m} \\
& \mathrm{E}=1.24 \times 10^{-2} \mathrm{eV}=12.4 \mathrm{meV}
\end{aligned}
$$

g) Visible

$$
\begin{aligned}
& \lambda=10^{-6} \mathrm{~m} \\
& \mathrm{E}=1.24 \mathrm{eV}
\end{aligned}
$$

h) Ultraviolet

$$
\begin{aligned}
& \lambda=10^{-7} \mathrm{~m} \\
& \mathrm{E}=12.4 \mathrm{eV}
\end{aligned}
$$

i) X-rays

$$
\begin{aligned}
& \lambda=10^{-10} \mathrm{~m} \\
& \mathrm{E}=1.24 \times 10^{4} \mathrm{eV}=12.4 \mathrm{keV}
\end{aligned}
$$

j) Gamma Rays

$$
\begin{aligned}
& \lambda=10^{-14} \mathrm{~m} \\
& \mathrm{E}=1.24 \times 10^{8} \mathrm{eV}=124 \mathrm{MeV}
\end{aligned}
$$

Energy of a photon indicates the amount of energy it needs to be emitted.
8.10. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of $2.0 \times 1010 \mathrm{~Hz}$ and amplitude 48 V m-1.
a) What is the wavelength of the wave?
b) What is the amplitude of the oscillating magnetic field?
c) Show that the average energy density of the $E$ field equals the average energy density of the $B$ field. [ $c=3 \times 108 \mathrm{~ms}-1$.]
Sol.

Given:
Frequency $=2 \times 10^{10} \mathrm{~Hz}$
Eo $=48 \mathrm{~V} / \mathrm{m}$
a) Wavelength is given by

$$
\begin{aligned}
& \lambda=\mathrm{c} / \mathrm{f} \\
& =300000000 / 20000000000 \\
& =0.015 \mathrm{~m}
\end{aligned}
$$

b) Using the relation
$\mathrm{Eo} / \mathrm{Bo}=\mathrm{c}$
$\mathrm{Bo}=\mathrm{Eo} / \mathrm{c}$

$$
\begin{aligned}
& \text { Or } \\
& \text { Bo }=48 / 300000000 \\
& =1.6 \times 10^{-7} \mathrm{~T}
\end{aligned}
$$

c) Average energy of electric field

$$
\mathrm{U}_{\mathrm{E}}=\frac{1}{2} c_{k} E^{2}
$$

Average energy of magnetic field

$$
\mathrm{U}_{\mathrm{B}}=\frac{1}{2 \mu_{0}} B^{2}
$$

Now E and B are related by the expression

$$
\mathrm{E}=\mathrm{c} . \mathrm{B}
$$

Substituting the relevant values in the above relation

$$
\sqrt{\frac{2 U_{E}}{c_{0}}}=\sqrt{\frac{1}{\mu_{0} c_{0}}} \cdot \sqrt{2 \mu_{0} U_{B}}
$$

Squaring both sides and cancelling the common terms, we get

$$
\mathrm{U}_{\mathrm{E}}=\mathrm{U}_{\mathrm{B}}
$$

Hence shown!

