Simplifying Test Prep

Chapter. 5<br>Magnetism and Matter<br>Class - XII<br>Subject - Physics

5.1. Answer the following questions regarding earth's magnetism:
a) A vector needs three quantities for its specification. Name thethree independent quantities conventionally used to specify the earth's magnetic field.

Sol.

The three quantities are: horizontal component of earth's magnetic field, magnetic declination and angle of dip.
b) The angle of dip at a location in southern India is about $\mathbf{1 8}^{\circ}$. Would you expect a greater or smaller dip angle in Britain?

Sol.

Great, because it increases from equator to poles.
c) If you made a map of magnetic field lines at Melbourne in Australia, would the lines seem to go into the ground or come out of the ground?

Sol.

Coming out of the ground.

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d) In which direction would a compass free to move in the vertical plane point to, if located right on the geomagnetic north or south pole?
Sol.

Compass can point in any direction at poles.
e) The earth's field, it is claimed, roughly approximates the field due to a dipole of magnetic moment $8 \times 1022 \mathrm{~J}$ T-1 located at its centre. Check the order of magnitude of this number in some way.

Sol.

By the formula:

$$
B=\frac{\mu_{o} m}{4 \pi r^{3}}
$$

Substituting $\mathrm{m}=8 \times 10^{22} \mathrm{~J} / \mathrm{T}$

$$
\mathrm{r}=6.4 \times 10^{6} \mathrm{~m}
$$

Thus

$$
\mathrm{B}=0.3 \mathrm{G}
$$

f) Geologist's claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing possible at all?

## Sol.

Because of local magnetic field due to magnetic deposits or heavy solar flares.

### 5.2. Answer the following questions:

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a) The earth's magnetic field varies from point to point in space. Does it also change with time? If so, on what time scale does it change appreciably?

Sol.

Yes it changes with time. The time scale is of order of few hundred years.
b) The earth's core is known to contain iron. Yet geologists do not regard this as a source of the earth's magnetism. Why?

Sol.
As molten iron is not ferromagnetic.
c) The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might be the 'battery' (i.e., the source of energy) to sustain these currents?

Sol.
The answer is uncertain. Radioactivity may be one reason.
d) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past?

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Sol.

Analysis of rock magnetism gives the clue.
e) The earth's field departs from its dipole shape substantially at large distances (greater than about $\mathbf{3 0 , 0 0 0} \mathbf{~ k m}$ ). What agencies may be responsible for this distortion?

Sol.

The extra activities like solar winds from space affect magnetism of earth.
f) Interstellar space has an extremely weak magnetic field of the order of 10-12 T. Can such a weak field be of any significant consequence? Explain.

Sol.

The interstellar distances are very great, and even the small minute field can affect the passage of charged particles.
5.3. A short bar magnet placed with its axis at $30^{\circ}$ with a uniform externalmagnetic field of 0.25 T experiences a torque of magnitude equal to $4.5 \times 10-2 \mathrm{~J}$. What is the magnitude of magnetic moment of the magnet?

Sol.
Given:
$\Theta=30^{\circ}$
$\mathrm{B}=0.25 \mathrm{~T}$
$\mathrm{T}=4.5 \times 10-2 \mathrm{~J}$

Torque is given by

$$
\mathrm{T}=\mathrm{mB} \cdot \sin \Theta
$$

Substitution yeilds

$$
4.5 \times 10-2=\mathrm{m} .(0.25) .(0.5)
$$

Therefore magnetic moment, $\mathrm{m}=0.36 \mathrm{~J} / \mathrm{T}$
5.4. A short bar magnet of magnetic moment $m=0.32 \mathrm{JT}-1$ is placed in auniform magnetic field of $\mathbf{0 . 1 5} \mathbf{T}$. If the bar is free to rotate in the plane of the field, which orientation would correspond to its
a) stable and
b) Unstable equilibrium?

What is the potential energy of the magnet in each case?

Sol.

Given:
$\mathrm{m}=0.32 \mathrm{~J} / \mathrm{T}$
$B=0.15 \mathrm{~T}$

The bar is free to rotate in the plane of the field. It will be in stable position in which the net torque is zero. Since the field is uniform, there will be no net force.

The angle between the field and the moment should be either zero or 180 degrees. This implies that it can be parallel or antiparallel to field.

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a) For stable equilibrium, magnetic moment m should be parallel to applied magnetic field.
b) For unstable equilibrium, magnetic moment should be antiparallel to applied magnetic field.
Now calculating potential energy of the magnet in each case.

For stable equilibrium

Potential energy

$$
\mathrm{U}=-\mathrm{mB}
$$

Putting the values

$$
\begin{aligned}
& U=-0.32 \times 0.15 \\
& U=-0.048 \mathrm{~J}
\end{aligned}
$$

For unstable equilibrium

$$
\begin{aligned}
\mathrm{U} & =\mathrm{mB} \\
& =0.32 \times 0.15 \\
& =+0.048 \mathrm{~J}
\end{aligned}
$$

5.5. A closely wound solenoid of $\mathbf{8 0 0}$ turns and area of cross section $2.5 \times 10-4 \mathbf{~ m} 2$ carries a current of $\mathbf{3 . 0} \mathrm{A}$. Explain the sense in which the solenoid acts like a bar magnet. What is its associated magnetic moment?

Sol.
Given:
$\mathrm{N}=800$
$\mathrm{A}=2.5 \times 10^{-4}$ sq. m
$\mathrm{I}=3 \mathrm{~A}$

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The solenoid acts like a bar magnet along the axis, determined by flow of current.
The magnetic moment associated

$$
\mathrm{m}=\mathrm{NIA}=0.6 \mathrm{~J} / \mathrm{T}
$$

### 5.6. If the solenoid in Exercise 5.5 is free to turn about the verticaldirection and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of torque on the solenoid when its axis makes an angle of $30^{\circ}$ with the direction of applied field?

Sol.

Added detail:
Horizontal magnetic field, $\mathrm{B}=0.25 \mathrm{~T}$
$\Theta=30$
Torque is experienced by the solenoid given by
Torque $=m B \sin \Theta$

$$
\begin{aligned}
& =0.6 \times 0.25 \times 0.5 \\
& =0.075 \mathrm{Nm}
\end{aligned}
$$

5.7. A bar magnet of magnetic moment 1.5 J T -1 lies aligned with thedirection of a uniform magnetic field of 0.22 T .
a) What is the amount of work required by an external torque to turn the magnet so as to align its magnetic moment:
i. normal to the field direction,
ii. Opposite to the field direction?
b) What is the torque on the magnet in cases (i) and (ii)?

Sol.

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Given:
$\mathrm{m}=1.5 \mathrm{~J} / \mathrm{T}$
$\mathrm{B}=0.22 \mathrm{~T}$
The magnet is aligned with the direction of uniform magnetic field, and the net force is zero.
a)
i. $\quad \Theta$ at normal to the field direction is 90 .

The work required is given by

$$
\mathrm{U}=\mathrm{mB}
$$

Substituting the values

$$
\begin{aligned}
\mathrm{U} & =1.5 \times 0.22 \\
\text { Or } \mathrm{U} & =0.33 \mathrm{~J}
\end{aligned}
$$

ii. Opposite to the field direction $=180$

So the work required will be double as calculated above.

Thus it is 0.66 J
b)The value of torque can be determined by the formula $T=m . B \cdot \sin \Theta$

Considering the both cases
i. $\quad \Theta=90$

$$
\begin{aligned}
\mathrm{T} & =\mathrm{mB} \sin 90 \\
& =1.5 \times 0.22 \times 1 \\
& =0.33 \mathrm{Nm}
\end{aligned}
$$

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ii. $\quad \Theta=180$

$$
\begin{aligned}
\mathrm{T} & =\mathrm{m} \mathrm{~B} \sin 180 \\
& =\text { zero } \mathrm{N}
\end{aligned}
$$

5.8. A closely wound solenoid of $\mathbf{2 0 0 0}$ turns and area of crosssection $1.6 \times 10-4 \mathrm{~m} 2$, carrying a current of 4.0 A , is suspended through its centre allowing it to turn in a horizontal plane.
a) What is the magnetic moment associated with the solenoid?
b) What is the force and torque on the solenoid if a uniform horizontal magnetic field of $7.5 \times 10-2 \mathrm{~T}$ is set up at an angle of $30^{\circ}$ with the axis of the solenoid?

Sol.
Given:

$$
\begin{aligned}
& \mathrm{N}=2000 \\
& \mathrm{~A}=1.6 \times 10^{-4} \mathrm{~m}^{2} \\
& \mathrm{I}=4 \mathrm{~A}
\end{aligned}
$$

a) Magnetic moment is given by

$$
\mathrm{m}=\mathrm{NIA}
$$

Solving by putting values
$\mathrm{m}=1.28 \mathrm{Am}^{2}$
b) The question mentions that the given magnetic field is uniform in nature. In uniform magnetic field, the net force is always zero, irrespective of the magnitude.
Now the torque on the solenoid is given by

$$
\mathrm{T}=\mathrm{m} \mathrm{~B} \sin \Theta
$$

Given $\mathrm{m}=1.28$ Am2 as calculated above

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$$
\begin{gathered}
B=0.075 \mathrm{~T} \\
\Theta=30
\end{gathered}
$$

Therefore
$\mathrm{T}=0.048 \mathrm{Nm}$
5.9. A circular coil of $\mathbf{1 6}$ turns and radius 10 cm carrying a current of0.75 A rests with its plane normal to an external field of magnitude $5.0 \times 10-2 \mathrm{~T}$. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of $2.0 \mathrm{~s}^{-1}$. What is the moment of inertia of the coil about its axis of rotation?

Sol.

Given in the question:
$\mathrm{N}=16$
$\mathrm{r}=10 \mathrm{~cm}$
$\mathrm{I}=0.75 \mathrm{~A}$
$\mathrm{B}=0.05 \mathrm{~T}$
$\Theta=90$
$\mathrm{f}=2 / \mathrm{s}$

The moment of inertia is described by the formula

$$
\begin{equation*}
I=\frac{m B}{4 \pi^{2} f^{2}} . \tag{1}
\end{equation*}
$$

Determining the value of magnetic moment

$$
\mathrm{m}=\mathrm{NIA}
$$

Or

$$
\mathrm{m}=16 \times 0.75 \times 3.14 \times 0.1 \times 0.1
$$

Or $\quad \mathrm{m}=0.377 \mathrm{~J} / \mathrm{T}$

Substituting the values in eq. (1)

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Moment of inertia of coil

$$
\mathrm{I}=1.19 \times 10^{-4} \mathrm{kgm}^{2}
$$

5.10. A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at $22^{\circ}$ with the horizontal. The horizontal component of the earth's magnetic field at the place is known to be 0.35 G . Determine the magnitude of the earth's magnetic field at the place.

Sol.

## Given:

Angle of dip, I = 22 degrees
$\mathrm{He}=0.35 \mathrm{G}$

Using the relation
$\cos 22=\mathrm{He} / \mathrm{Be}$
$\mathrm{Be}=0.35 \mathrm{G} / \cos 22=0.377 \mathrm{G}$
5.11. At a certain location in Africa, a compass points $12^{\circ}$ west of thegeographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points $60^{\circ}$ above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G . Specify the direction and magnitude of the earth's field at the location.

Sol.

Data provided:
I = 60 degrees
$\mathrm{He}=0.16 \mathrm{G}$

Using the approach of previous question

$$
\begin{aligned}
\mathrm{Be} & =0.16 \mathrm{G} / \cos 60 \\
& =0.32 \mathrm{G}
\end{aligned}
$$

Direction: magnetic north to magnetic south
5.12. A short bar magnet has a magnetic moment of $\mathbf{0 . 4 8} \mathbf{J} \mathbf{T}-\mathbf{1}$. Give thedirection and magnitude of the magnetic field produced by the magnetat a distance of 10 cm from the centre of the magnet on
a) the axis,
b) the equatorial lines (normal bisector) of the magnet.

Sol.
Given:

$$
\begin{aligned}
& \mathrm{m}=0.48 \mathrm{~J} / \mathrm{T} \\
& \mathrm{r}=10 \mathrm{~cm}
\end{aligned}
$$

The magnet is short. This indicates that the length of magnet is insignificant when compared to the distance at which the magnitude and direction of magnetic field is to be determined.
$\mathrm{r} \gg l$
The magnitude of the magnetic field at a distance of 10 cm by magnet will be different for both the cases. The direction in both the cases will be different too.
Calculating for the cases:

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a) On the axis

Magnetic field is defined by the formula

$$
B=\frac{\mu_{o} 2 m}{4 \pi r^{3}}
$$

Substitution yields
B $=9.6 \times 10^{-5} \mathrm{~T}$
Direction: South to North
b) On normal bisector

Magnetic field is defined by

$$
B=\frac{\mu_{o} m}{4 \pi r^{3}}
$$

Solving
$B^{\prime}=4.8 \times 10^{-5} \mathrm{~T}$
Direction: North to South
5.13. A short bar magnet placed in a horizontal plane has its axis alignedalong the magnetic north-south direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0.36 G and the angle of dip is zero. What is the total magnetic field on the normal bisector of the magnet at the same distance as the null-point (i.e., 14 cm ) from the centre of the magnet? (At null points, field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field.)

Sol.
Given:

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$\mathrm{Be}=0.36 \mathrm{G}$
I = 0 degree
Null point at $\mathrm{r}=14 \mathrm{~cm}$

Since the given magnet is short, the length of the magnet plays no significant role in our calculations. The magnet is aligned along the earth's magnetic field.

Determining the horizontal component of magnetic field
$\mathrm{He} / \mathrm{Be}=\cos 0$
Or $\mathrm{He} / \mathrm{Be}=1$
Or $\mathrm{He}=\mathrm{Be}=0.36 \mathrm{G}$

To calculate the magnetic field at normal bisector of magnet, we have to determine the magnetic moment of magnet first.

Magnetic field due to magnet at its axis at null point, $\mathrm{B}^{\prime}=\frac{\mu_{o} 2 m}{4 \pi r^{3}}$
This magnetic field will be equal and opposite to horizontal component of earth's magnetic field at null point. So
$\mathrm{B}^{\prime}=\mathrm{He}$
Or $\quad\left(10^{-7}\right) \cdot(2 \mathrm{~m}) /(0.14)^{3}=0.36 \times 10^{-4}$
This gives

$$
\mathrm{m}=0.49392 \mathrm{~J} / \mathrm{T}
$$

Now determining the magnetic field at normal bisector of magnet at given distance of $\mathrm{r}=14 \mathrm{~cm}$
$\mathrm{B}=\frac{\mu_{o} m}{4 \pi r^{3}}$
which yields

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$B=0.18 G$
Now
Total Magnetic field
$=$ magnetic field due to magnet + magnetic field due to earth Therefore Total Magnetic field $=0.18 \mathrm{G}+0.36 \mathrm{G}=0.54 \mathrm{G}$
Direction: In the direction of earth's field.
5.14. If the bar magnet in exercise 5.13 is turned around by $180^{\circ}$, wherewill the new null points be located?

Sol.

New null point at normal bisector

$$
\mathrm{B}=\frac{\mu_{o} m}{4 \pi r^{3}}
$$

It will be equal and opposite to horizontal component of earth's magnetic field at that point.
We have already determined magnetic moment in the previous question. Using that value

$$
\left(10^{-7}\right) \cdot(0.4939) /\left(0.36 \times 10^{-4}\right)=r^{3}
$$

Or

$$
\mathrm{r}=0.11 \mathrm{~m}
$$

Thus the null point will be at a distance of $r=11.1 \mathrm{~cm}$ on the normal bisector of the given magnet.
5.15. A short bar magnet of magnetic moment $5.25 \times 10-2 \mathrm{~J} \mathrm{~T}^{-1}$ is placedwith its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at $45^{\circ}$ with earth's field on
a) its normal bisector and
b) its axis.

Simplifying Test Prep Magnitude of the earth's field at the place is given to be 0.42 G . Ignore the length of the magnet in comparison to the distances involved.

Sol.

Given:
$\mathrm{m}=5.25 \times 10^{-2} \mathrm{~J} / \mathrm{T}$
$\Theta=90$ degrees
$B=0.42 G$
a) At normal bisector

$$
\mathrm{B}=\frac{\mu_{o} m}{4 \pi r^{3}}
$$

Putting the values and solving for $r$
$\mathrm{r}=0.05 \mathrm{~m}$
Or $r=5 \mathrm{~cm}$
b) At axis

$$
\mathrm{B}=\frac{\mu_{o} 2 m}{4 \pi r^{3}}
$$

Solving similarly
$r^{\prime}=0.063 \mathrm{~m}$
Or $r^{\prime}=6.3 \mathrm{~cm}$

