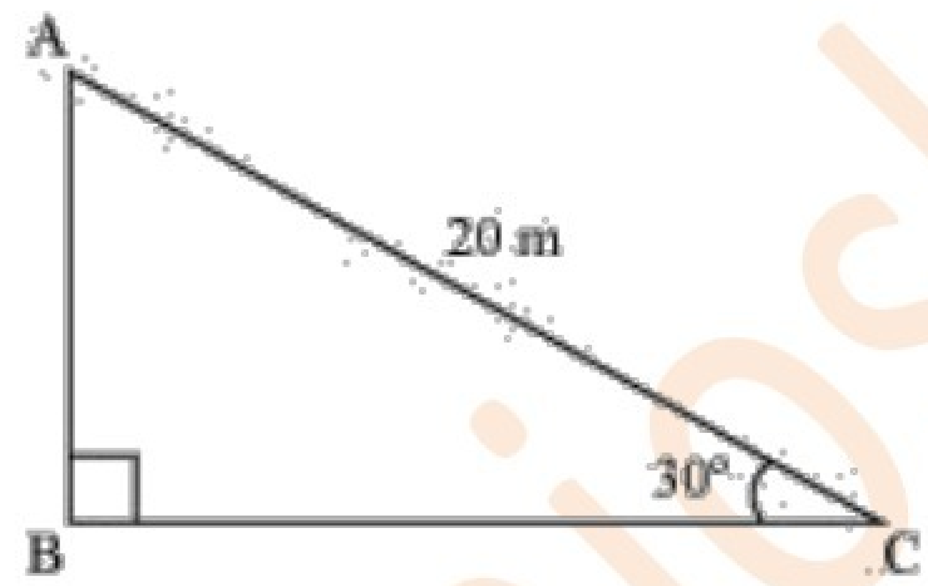
Chapter 9
Some Applications of Trigonometry

Exercise: 9.1

Question 1: A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30 °.



Solution:

It can be observed from the figure that AB is the pole.

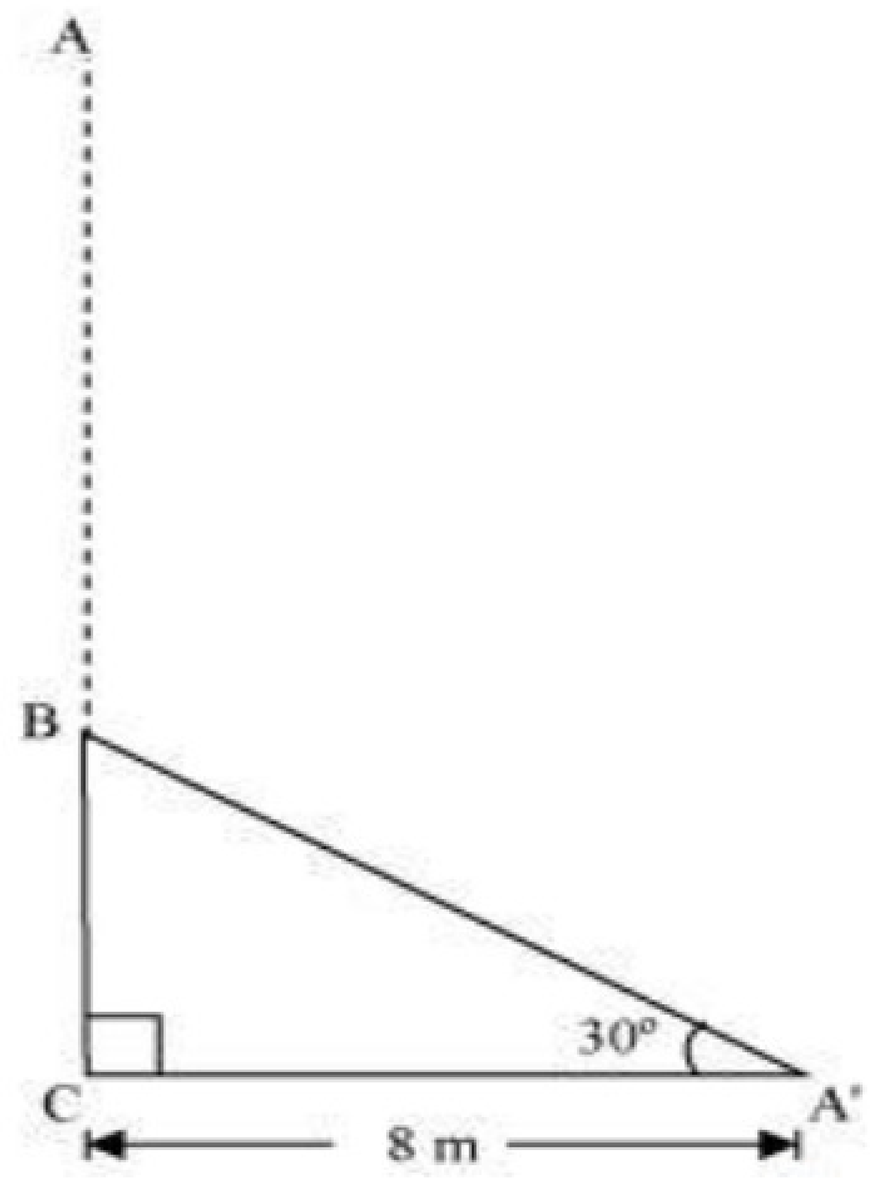
In ΔABC,

$$\frac{AB}{AC} = \sin 30^\circ$$
$$\frac{AB}{20} = \frac{1}{2}$$
$$AB = 10$$

Therefore, the height of the pole is 10 m.

Question 2: A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30 ° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Solution:



Let AC was the original tree. Due to storm, it was broken into two parts. The broken part A'B is making 30° with the ground.

In $\Delta A'BC$

$$\frac{BC}{A'C} = \tan 30^\circ$$

$$\frac{BC}{8} = \frac{1}{\sqrt{3}}$$

$$BC = \frac{8}{\sqrt{3}} m$$

$$\frac{A'C}{A'B} = \cos 30^\circ$$

$$\frac{8}{A'B} = \frac{\sqrt{3}}{2}$$

$$A'B = \frac{16}{\sqrt{3}} m$$

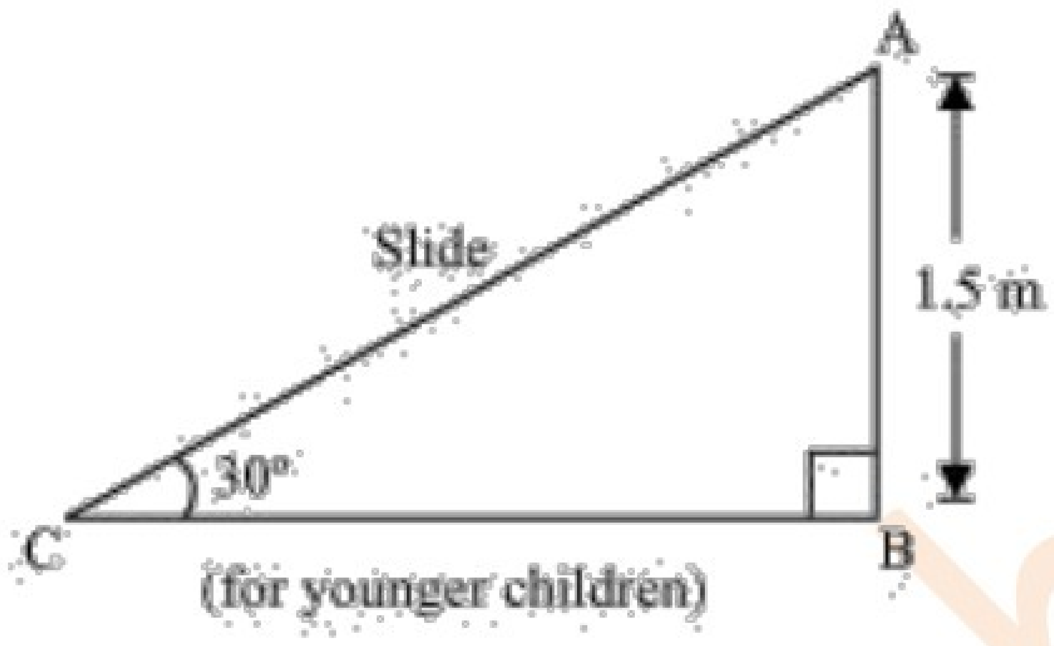
Height of tree = A'B+ BC

$$= \frac{8}{\sqrt{3}} m + \frac{16}{\sqrt{3}} m$$

$$= 8\sqrt{3} m$$

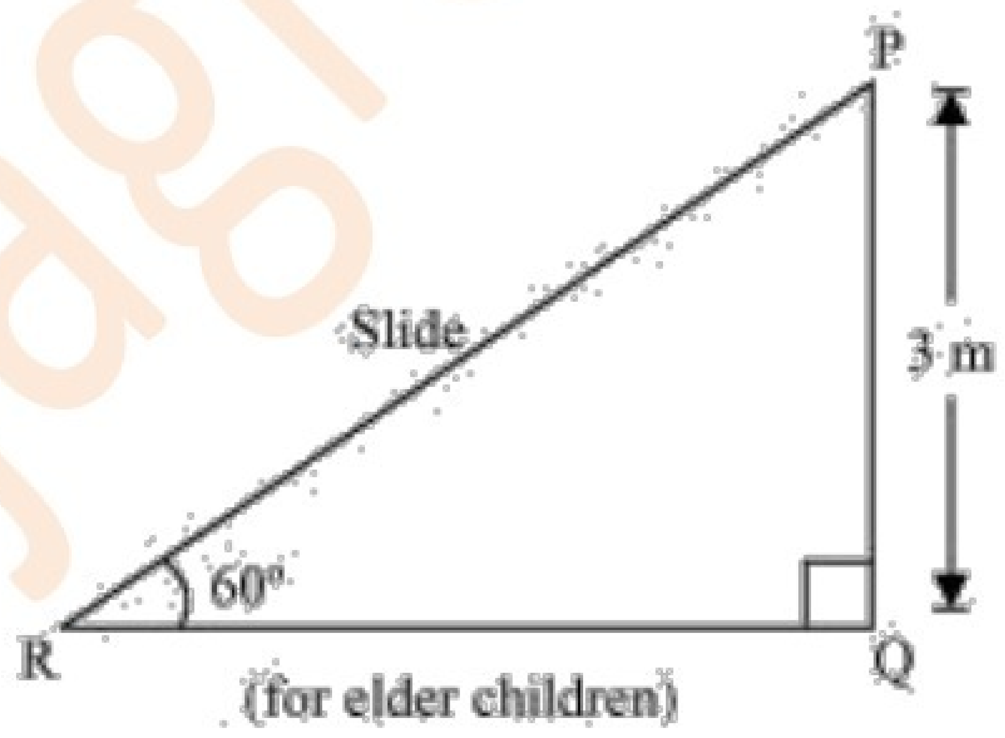
Question 3: A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30 ° to the ground, where as for the elder children she wants to have a steep side at a height of 3 m, and inclined at an angle of 60 ° to the ground. What should be the length of the slide in each case?

Solution: It can be observed that AC and PR are the slides for younger and elder children respectively.



In ΔABC,

$$\frac{AB}{AC} = \sin 30^\circ$$
$$\frac{1.5}{AC} = \frac{1}{2}$$
$$AC = 3$$



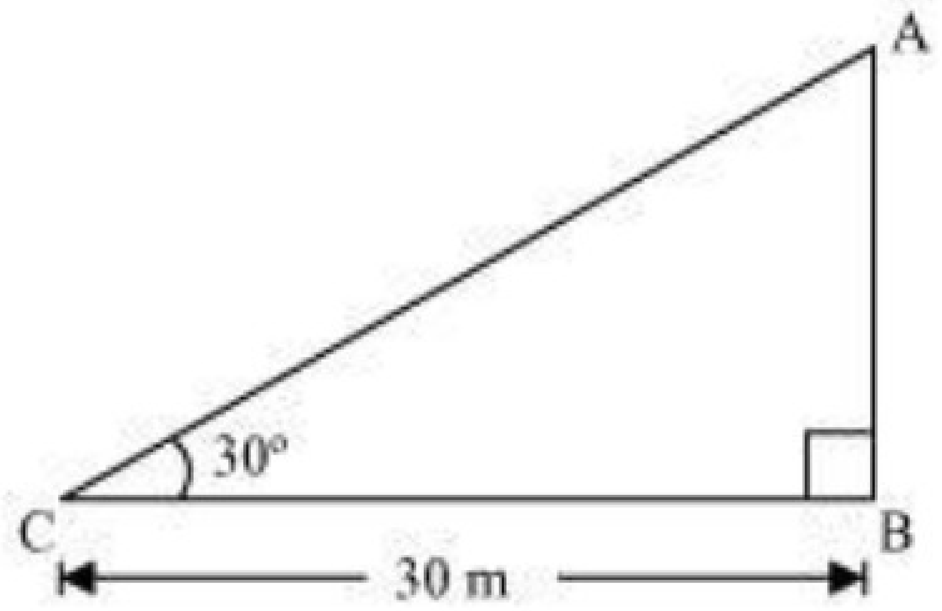
In ΔPQR,

$$\frac{PQ}{PR} = \sin 60^\circ$$
$$\frac{3}{PR} = \frac{\sqrt{3}}{2}$$
$$PR = 2\sqrt{3}$$

Therefore, the lengths of these slides are 3 m and $2\sqrt{3}m$.

Question 4: The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

Solution:



Let AB be the tower and the angle of elevation from point C (on ground) is 30°.

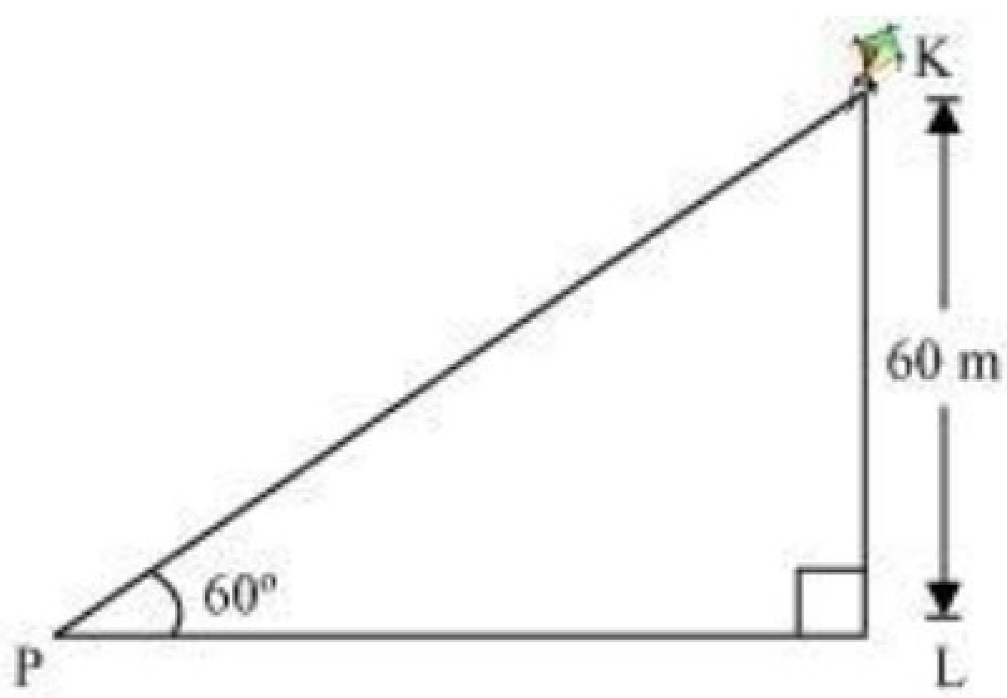
In ΔABC,

$$\frac{AB}{BC} = \tan 30^\circ$$
$$\frac{AB}{30} = \frac{1}{\sqrt{3}}$$
$$AB = 10\sqrt{3}$$

Therefore, the height of the tower is $10\sqrt{3}$.

Question 5: A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Solution:



Let K be the kite and the string is tied to point P on the ground.

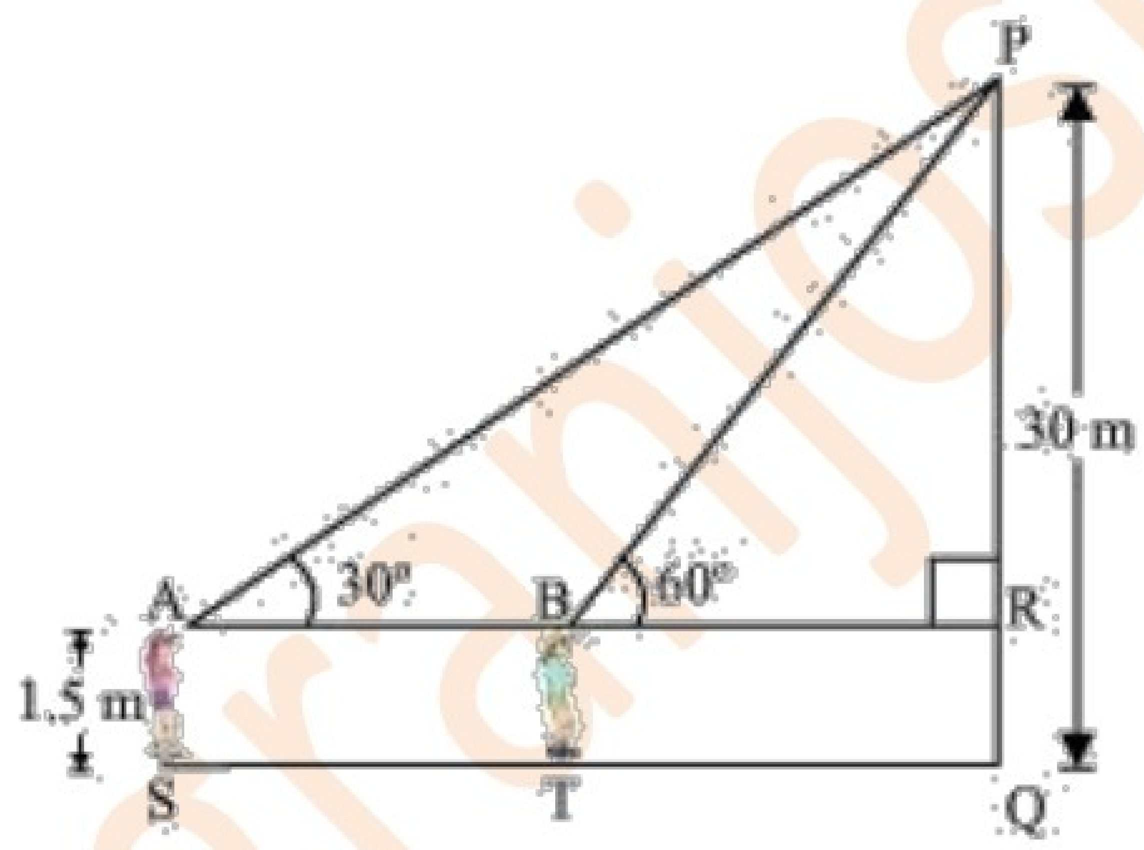
In ΔKLP,

$$\frac{KL}{KP} = \sin 60^\circ$$
$$\frac{60}{KP} = \frac{\sqrt{3}}{2}$$
$$KP = 40\sqrt{3}$$

Hence, the length of the string is $40\sqrt{3}$.

Question 6: A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Solution:



Let the boy was standing at point S initially. He walked towards the building and reached at point T.

It can be observed that

PR = PQ − RQ

$$= (30 - 1.5)\text{ m} = 28.5\text{ m} = \frac{57}{2} m$$

In ΔPAR,

$$\frac{PR}{AR} = \tan 30^\circ$$
$$\frac{57}{2AR} = \frac{1}{\sqrt{3}}$$
$$AR = \frac{57\sqrt{3}}{2} m$$

In ΔPRB,

$$\frac{PR}{BR} = \tan 60^\circ$$
$$\frac{57}{2BR} = \sqrt{3}$$
$$BR = \frac{19\sqrt{3}}{2} m$$

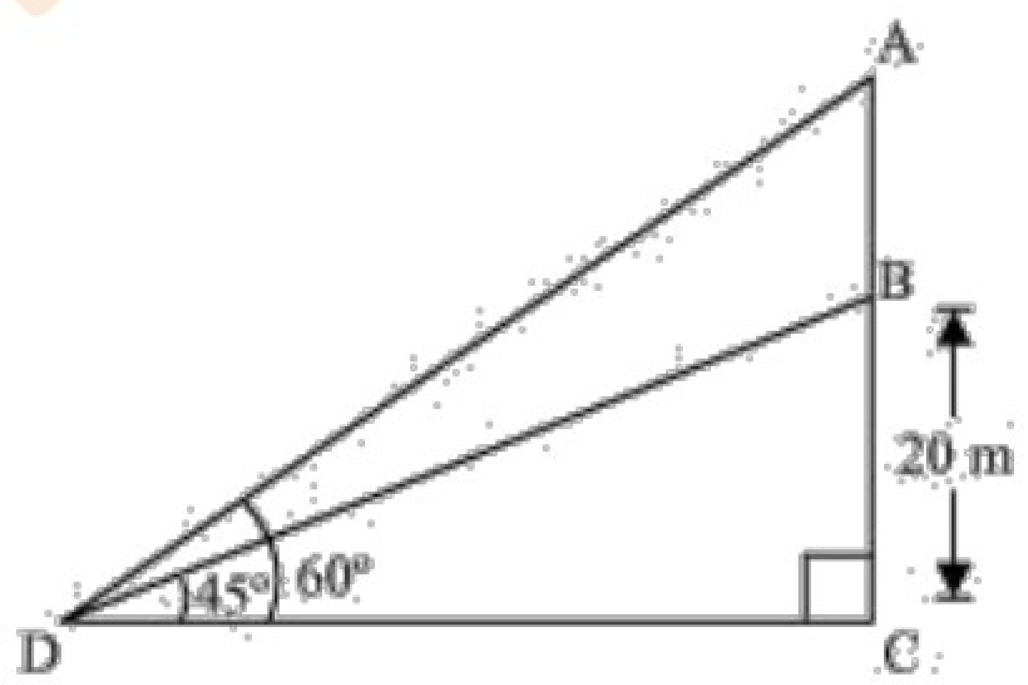
ST = AB

$$= AR - BR$$
$$= \frac{57\sqrt{3}}{2} - \frac{19\sqrt{3}}{2}$$
$$= \frac{38\sqrt{3}}{2}$$
$$= 19\sqrt{3} m$$

Hence, he walked $= 19\sqrt{3}m$ towards the building.

Question 7: From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Solution:



Let BC be the building, AB be the transmission tower, and D be the point on the ground from where the elevation angles are to be measured.

In ΔBCD,

$$\frac{BC}{CD} = \tan 45^\circ$$
$$\frac{20}{CD} = 1$$
$$CD = 20m$$

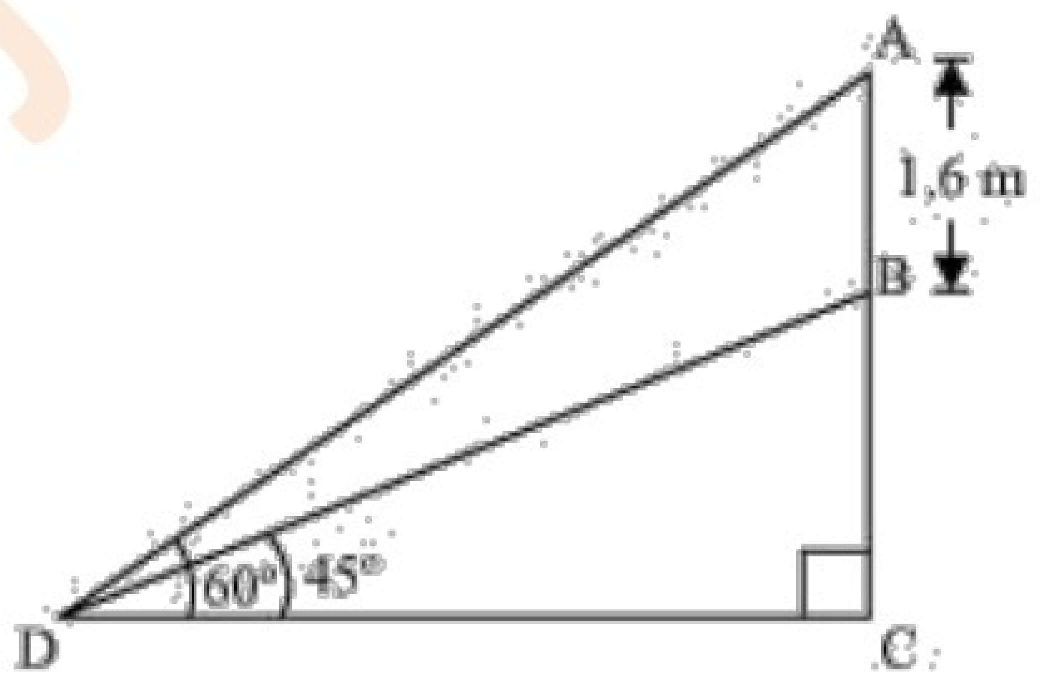
In ΔACD,

$$\frac{AC}{CD} = \tan 60^\circ$$
$$\frac{AB + BC}{CD} = \sqrt{3}$$
$$\frac{AB + 20}{20} = \sqrt{3}$$
$$AB = 20\left(\sqrt{3} - 1\right)m$$

Therefore, the height of the transmission tower is $20\left(\sqrt{3} - 1\right)m$.

Question 8: A statue, 1.6 m tall, stands on a top of pedestal, from a point on the ground, the angle of elevation of the top of statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45 °. Find the height of the pedestal.

Solution:



Let AB be the statue, BC be the pedestal, and D be the point on the ground from where the elevation angles are to be measured.

In ΔBCD,

$$\frac{BC}{CD} = \tan 45^\circ$$
$$\frac{BC}{CD} = 1$$
$$BC = CD$$

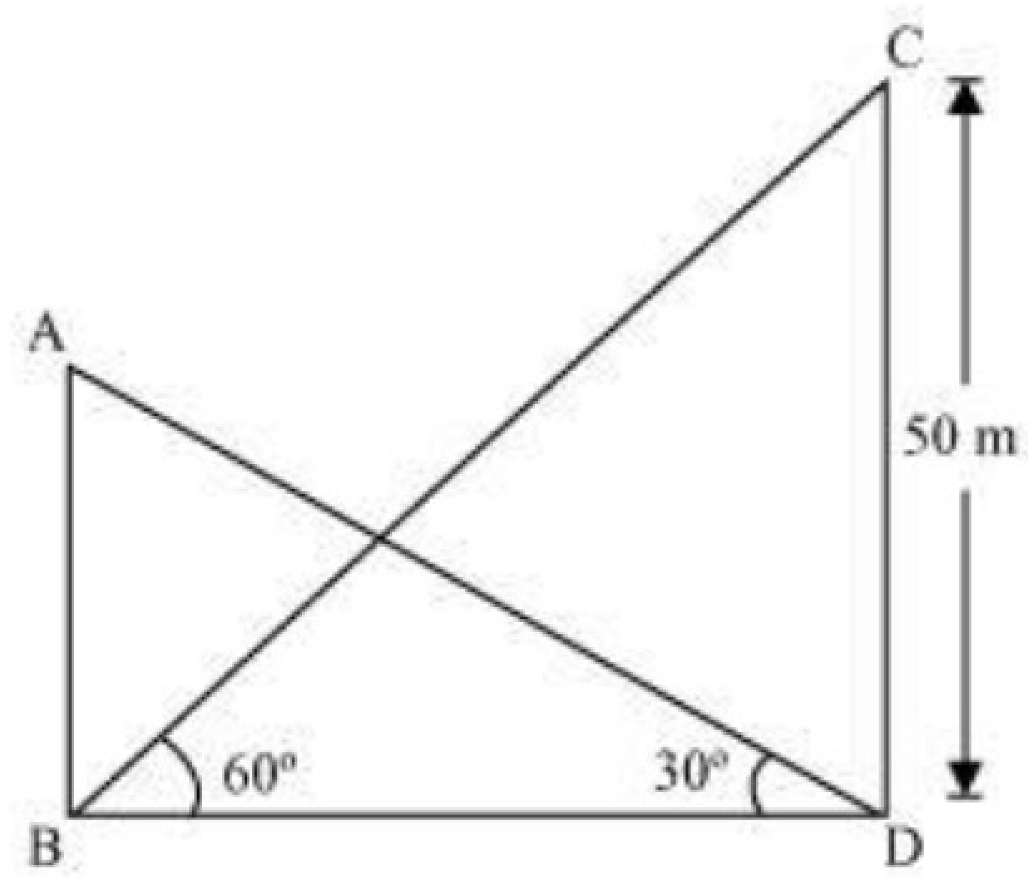
In ΔACD,

$$\frac{AB+BC}{CD} = \tan 60^\circ$$
$$\frac{AB+BC}{BC} = \sqrt{3}$$
$$1.6 + BC = BC\sqrt{3}$$
$$BC = \frac{1.6}{\sqrt{3}-1}$$
$$BC = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$
$$BC = 0.8\left(\sqrt{3}+1\right)$$

Therefore, the height of the pedestal is $0.8\left(\sqrt{3}+1\right)$ m.

Question 9: The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Solution:



Let AB be the building and CD be the tower.

In ΔCDB,

$$\frac{CD}{BD} = \tan 60^\circ$$
$$\frac{50}{BD} = \sqrt{3}$$
$$BD = \frac{50}{\sqrt{3}}$$

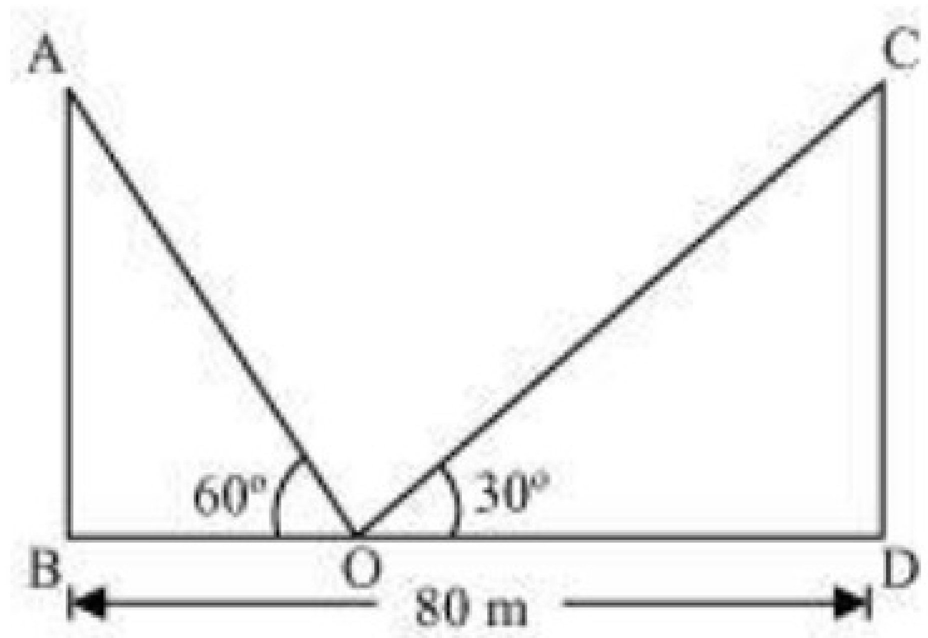
In ΔABD,

$$\frac{AB}{BD} = \tan 30^\circ$$
$$\frac{AB}{BD} = \frac{1}{\sqrt{3}}$$
$$AB = \frac{50}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}$$
$$AB = \frac{50}{3} m$$
$$AB = 16\frac{2}{3} m$$

Therefore, the height of the building is $16\frac{2}{3} m$.

Question 10: Two poles of equal heights are standing opposite each other an either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30º, respectively. Find the height of poles and the distance of the point from the poles.

Solution:



Let AB and CD be the poles and O is the point from where the elevation angles are measured.

In ΔABO,

$$\frac{AB}{BO} = \tan 60^\circ$$
$$\frac{AB}{BO} = \sqrt{3}$$
$$BO = \frac{AB}{\sqrt{3}}$$

In ΔCDO,

$$\frac{CD}{DO} = \tan 30^\circ$$
$$\frac{CD}{80 - BO} = \frac{1}{\sqrt{3}}$$
$$CD\sqrt{3} = 80 - BO$$
$$CD\sqrt{3} = 80 - \frac{AB}{\sqrt{3}}$$
$$CD\sqrt{3} + \frac{AB}{\sqrt{3}} = 80$$

Since the poles are of equal heights,

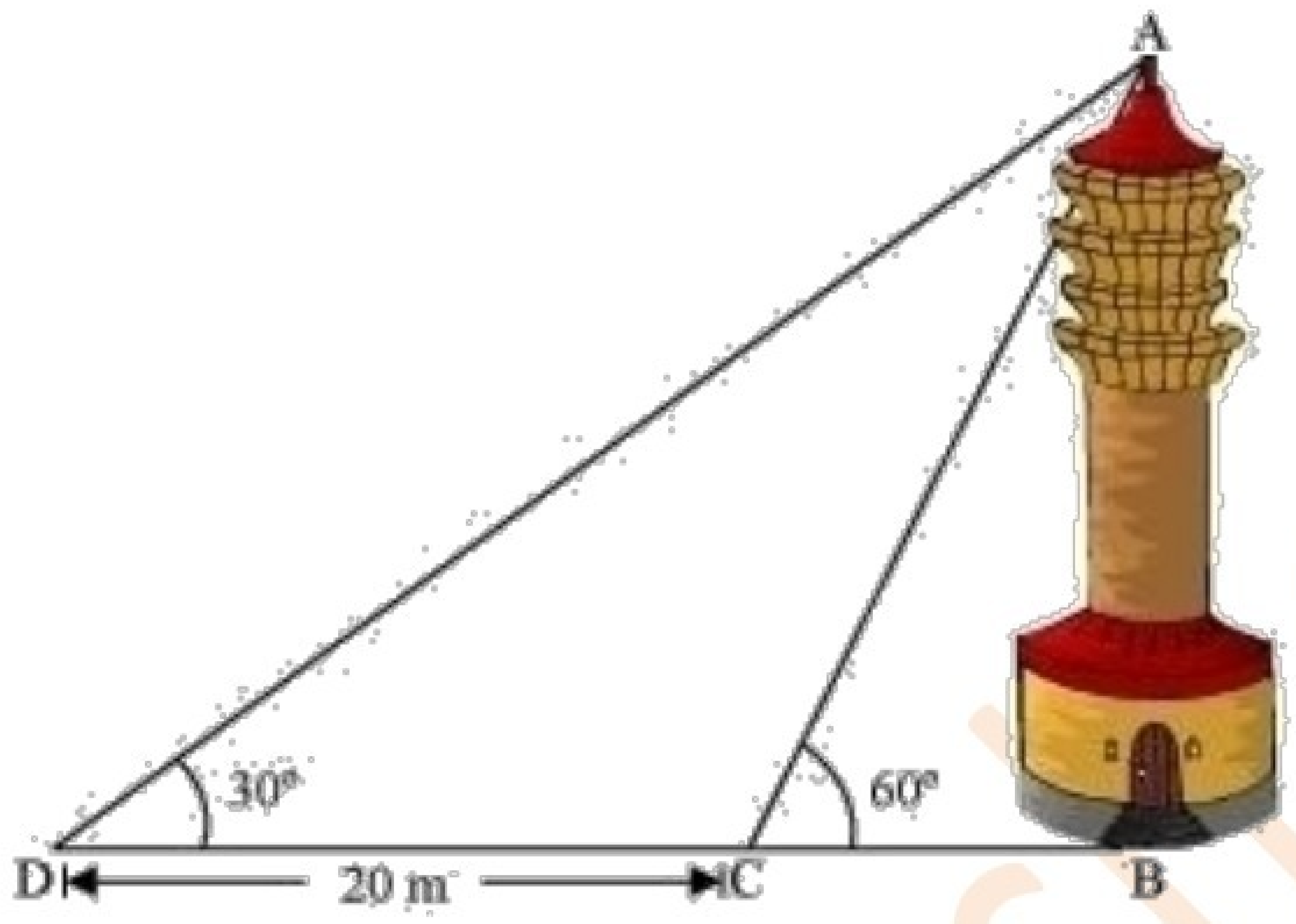
$$CD = AB$$

$$CD\sqrt{3} + \frac{CD}{\sqrt{3}} = 80$$
$$CD\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) = 80$$
$$CD\left(\frac{4}{\sqrt{3}}\right) = 80$$
$$CD = 20\sqrt{3}m$$
$$BO = \frac{AB}{\sqrt{3}} = \frac{CD}{\sqrt{3}} = 20m$$

$$DO = BD - BO = (80 - 20)\ m = 60\ m$$

Therefore, the height of poles is $20\sqrt{3}m$ and the point is 20 m and 60 m far from these poles.

Question 11: A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.



Solution:

In ΔABC,

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{AB}{BC} = \sqrt{3}$$

$$BC = \frac{AB}{\sqrt{3}}$$

In ΔABD,

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{AB}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\frac{AB}{\frac{AB}{\sqrt{3}} + 20} = \frac{1}{\sqrt{3}}$$

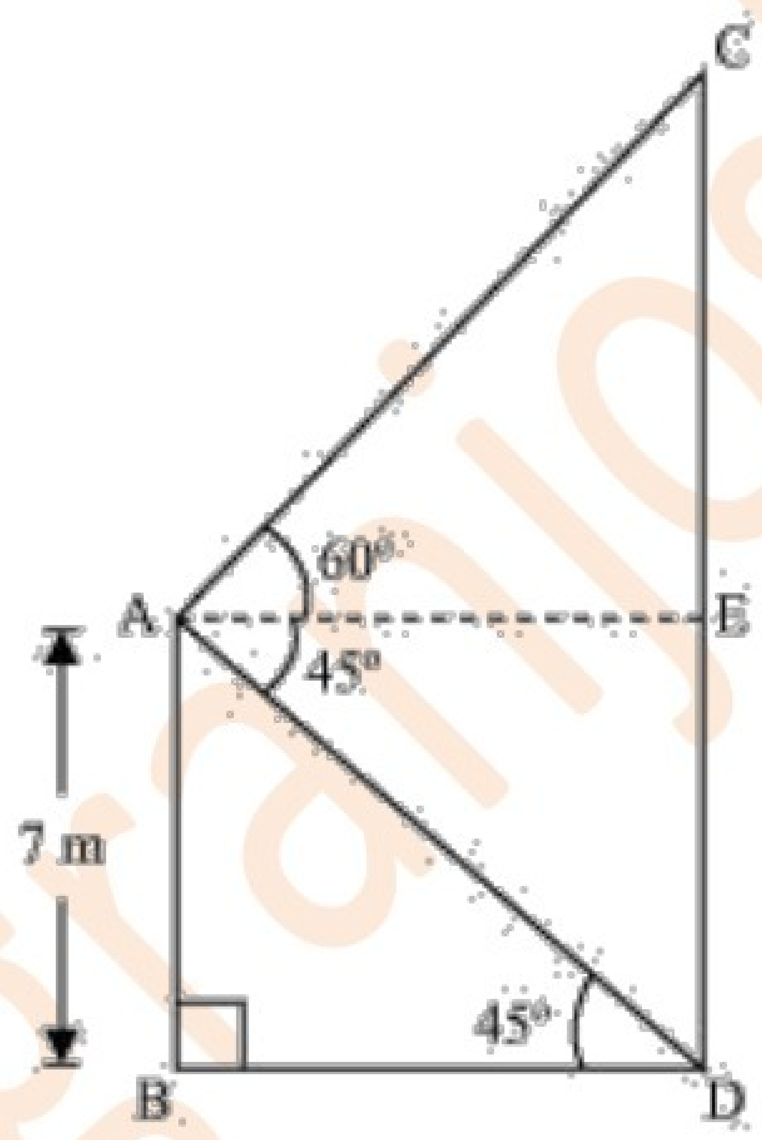
$$\frac{AB\sqrt{3}}{AB + 20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$3AB = AB + 20\sqrt{3}$$
$$2AB = 20\sqrt{3}$$
$$AB = 10\sqrt{3}$$
$$BC = 10m$$

Therefore, the height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m.

Question 12: From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Solution:



Let AB be a building and CD be a cable tower.

In ΔABD,

$$\frac{AB}{BD} = \tan 45^\circ$$
$$\frac{7}{BD} = 1$$
$$BD = 7m$$

In ΔACE,

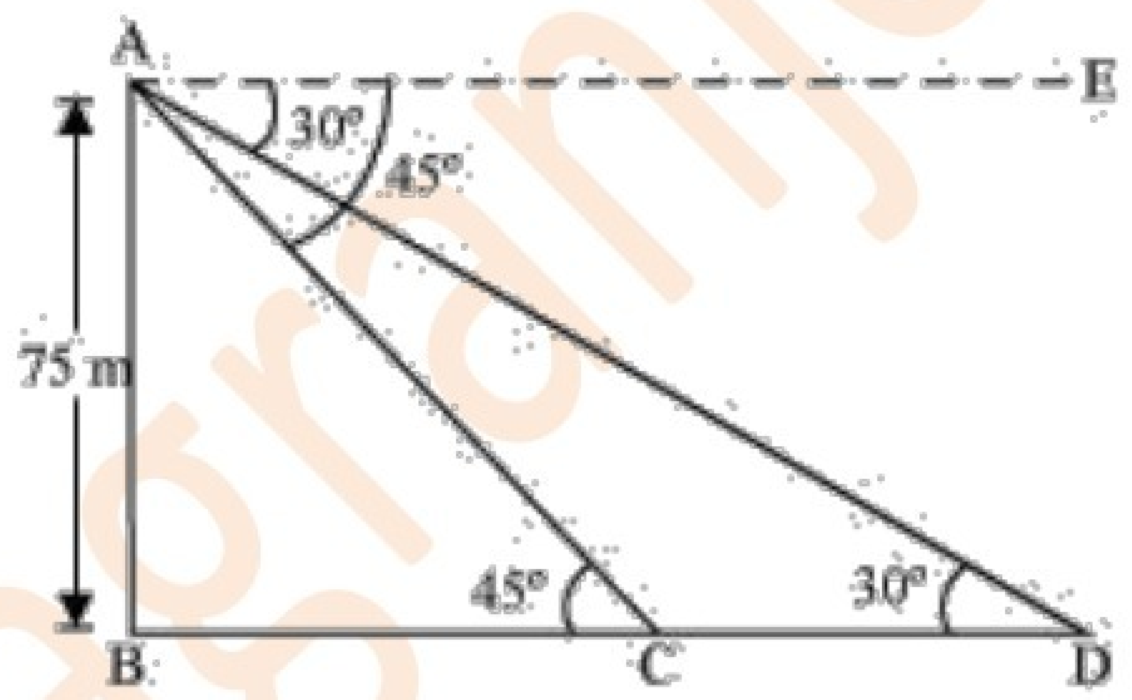
$$AE = BD = 7 \text{ m}$$

$$\frac{CE}{AE} = \tan 60^\circ$$
$$\frac{CE}{7} = \sqrt{3}$$
$$CE = 7\sqrt{3}m$$
$$CD = 7\sqrt{3} + 7$$
$$CD = 7\left(\sqrt{3}+1\right)m$$

Therefore, the height of the cable tower is $7\left(\sqrt{3}+1\right)m$.

Question 13: As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Solution:



Let AB be the lighthouse and the two ships be at point C and D respectively.

In ΔABC,

$$\frac{AB}{BC} = \tan 45^\circ$$
$$\frac{75}{BC} = 1$$
$$BC = 75m$$

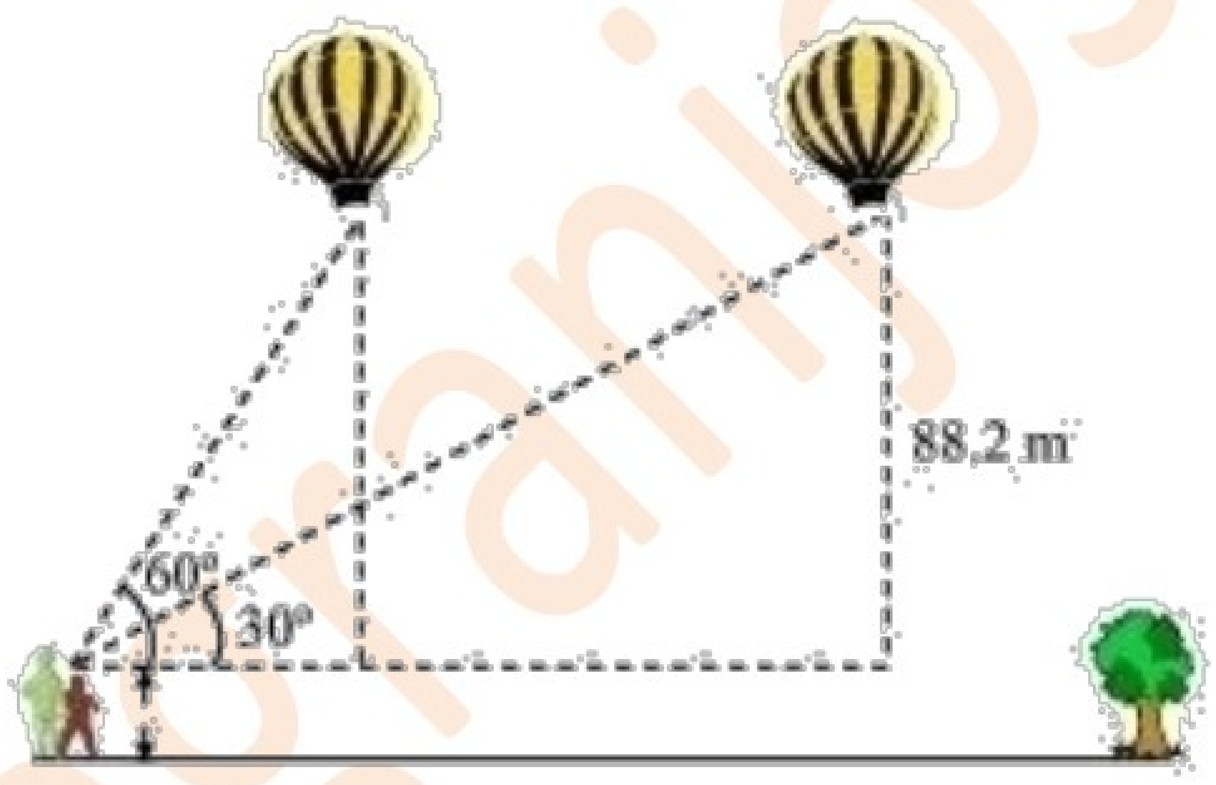
In ΔABD,

$$\frac{AB}{BD} = \tan 30^\circ$$

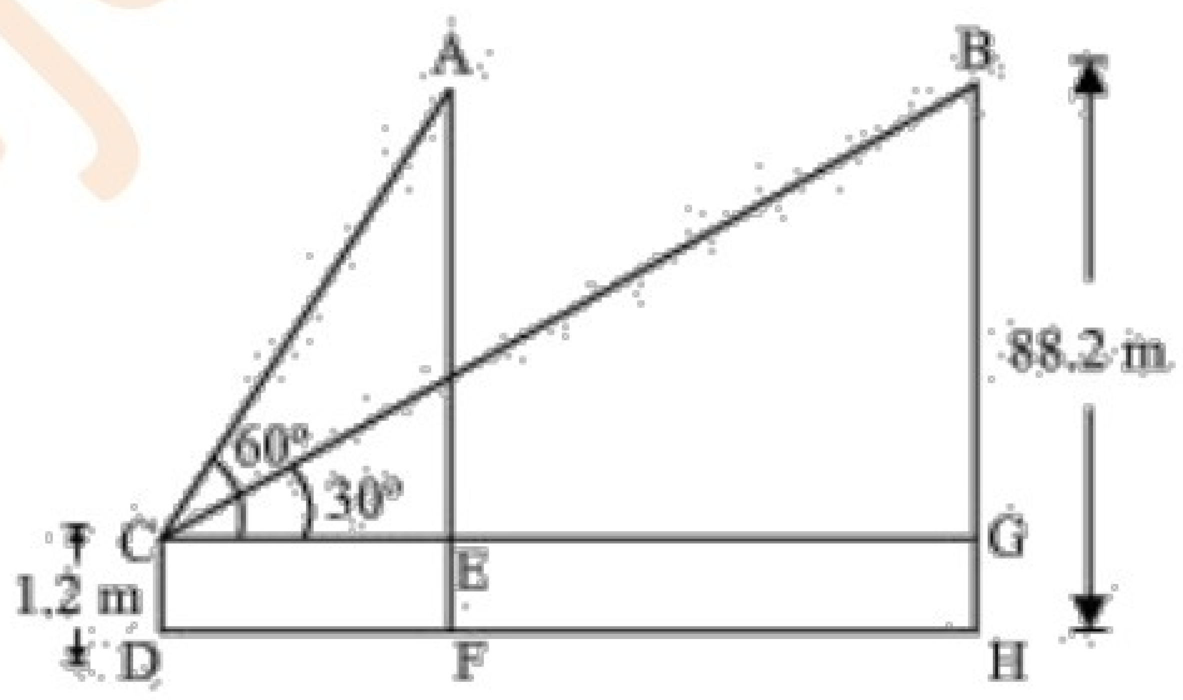
$$\frac{75}{BC+CD}=\frac{1}{\sqrt{3}}$$
$$\frac{75}{75+CD}=\frac{1}{\sqrt{3}}$$
$$75\sqrt{3}=75+CD$$
$$CD=75\left(\sqrt{3}-1\right)m$$

Therefore, the distance between the two ships is $75\left(\sqrt{3}-1\right)m$.

Question 14: A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.



Solution:



Let the initial position A of balloon change to B after some time and CD be the girl.

In ΔACE,

$$\frac{AC}{CE}=\tan 60^{\circ}$$

$$\frac{AF - EF}{CE} = \sqrt{3}$$

$$\frac{88.2 - 1.2}{CE} = \sqrt{3}$$

$$\frac{87}{CE} = \sqrt{3}$$

$$CE = 29\sqrt{3}m$$

In ΔBCG,

$$\frac{BC}{CG} = \tan 30^\circ$$

$$\frac{88.2 - 1.2}{CG} = \frac{1}{\sqrt{3}}$$

$$\frac{87}{CG} = \frac{1}{\sqrt{3}}$$

$$CG = 87\sqrt{3}m$$

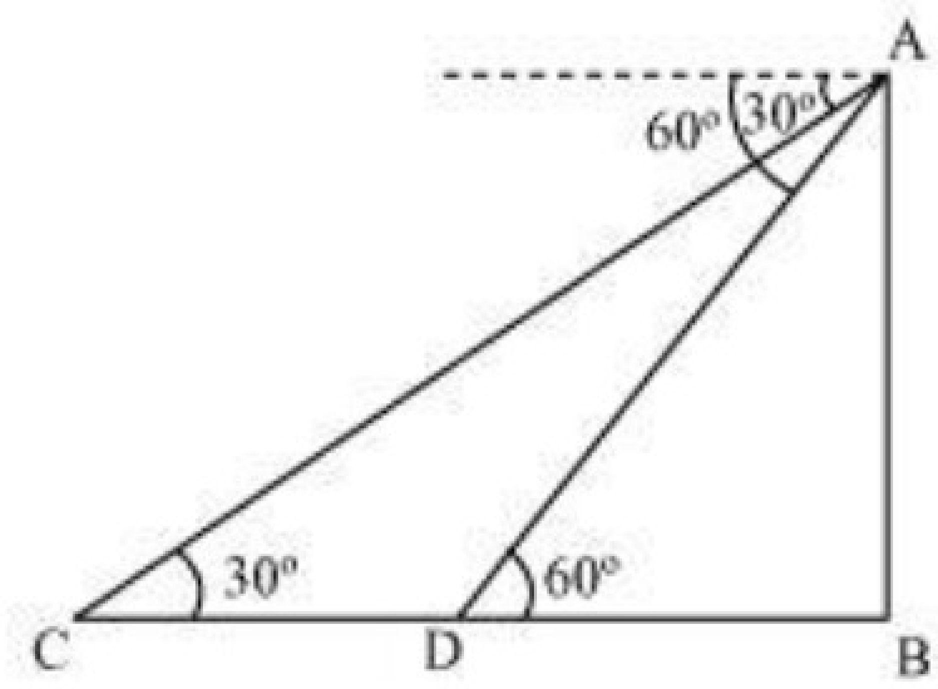
Distance travelled by balloon, $EG = CG - CE$

$$EG = 87\sqrt{3} - 29\sqrt{3}$$

$$EG = 58\sqrt{3}m$$

Question 15: A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car as an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Solution:



Let AB be the tower.

Initial position of the car is C, which changes to D after six seconds.

In ΔADB,

$$\frac{AB}{DB} = \tan 60^\circ$$
$$\frac{AB}{DB} = \sqrt{3}$$

$$DB = \frac{AB}{\sqrt{3}}$$

In ΔABC,

$$\frac{AB}{BC} = \tan 30^\circ$$
$$\frac{AB}{BD + DC} = \frac{1}{\sqrt{3}}$$
$$AB\sqrt{3} = BD + DC$$
$$AB\sqrt{3} = \frac{AB}{\sqrt{3}} + DC$$
$$DC = \frac{2AB}{\sqrt{3}}$$

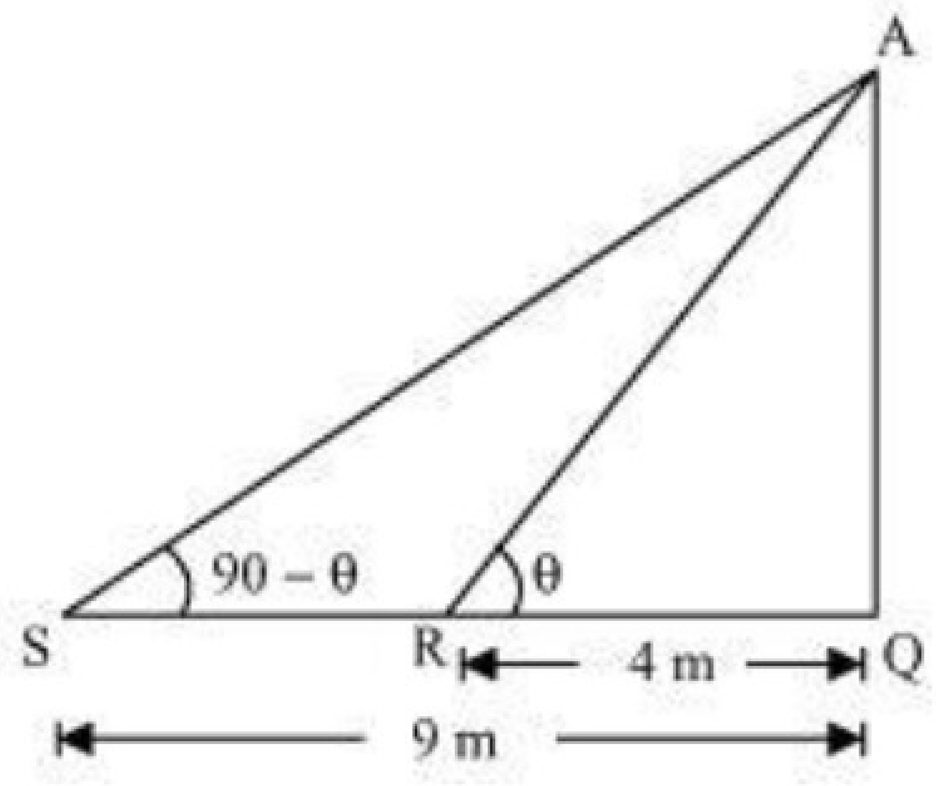
Time taken by the car to travel distance $DC = \frac{2AB}{\sqrt{3}} = 6$ seconds

$$AB = 3\sqrt{3}$$

Time taken by the car to travel distance DB $DB = \frac{AB}{\sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{3}} = 3$ seconds

Question 16: The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m. from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Solution:



Let AQ be the tower and R, S are the points 4m, 9m away from the base of the tower respectively.

The angles are complementary. Therefore, if one angle is θ, the other will be 90 − θ.

In ΔAQR,

$$\frac{AQ}{QR} = \tan\theta$$

$$\frac{AQ}{4} = \tan\theta \qquad \ldots(i)$$

In ΔAQS,

$$\frac{AQ}{SQ} = \tan\left(90^\circ - \theta\right)$$

$$\frac{AQ}{9} = \cot\theta \qquad \ldots(ii)$$

On multiplying equations (i) and (ii), we obtain

$$\left(\frac{AQ}{4}\right)\left(\frac{AQ}{9}\right) = \tan\theta.\cot\theta$$

$$AQ^2 = 36$$

$$AQ = \pm 6$$

However, height cannot be negative.

Therefore, the height of the tower is 6 m.