Chapter 8
Introduction to Trigonometry

Exercise: 8.1

Question 1: In ΔABC right angled at B, AB = 24 cm, BC = 7 m. Determine

(i) sin A, cos A
(ii) sin C, cos C

Solution:

Applying Pythagoras theorem for ΔABC, we obtain

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = (24\text{ cm})^2 + (7\text{ cm})^2$$

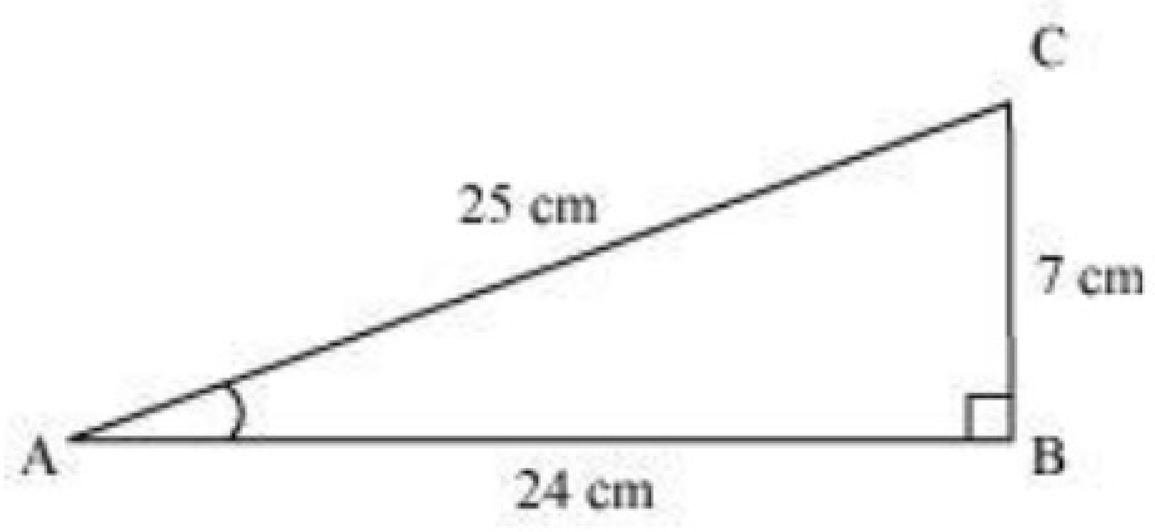
$$AC^2 = (576 + 49)\text{ cm}^2$$

$$AC^2 = 625\text{ cm}^2$$

$$AC = 25\text{ cm}$$

(i) $$\sin A = \frac{BC}{AC} = \frac{7}{25}$$

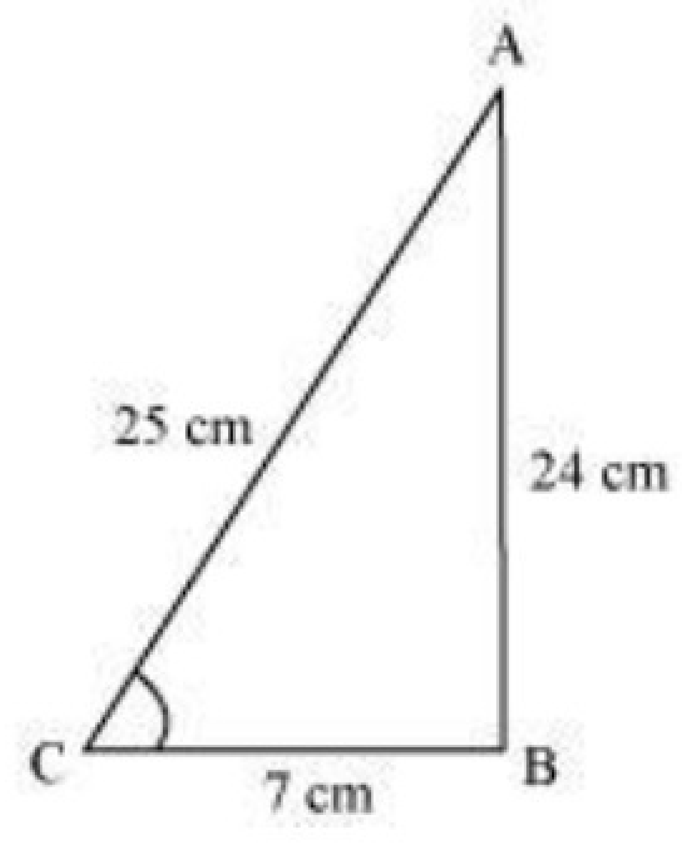
$$\cos A = \frac{AB}{AC} = \frac{24}{25}$$



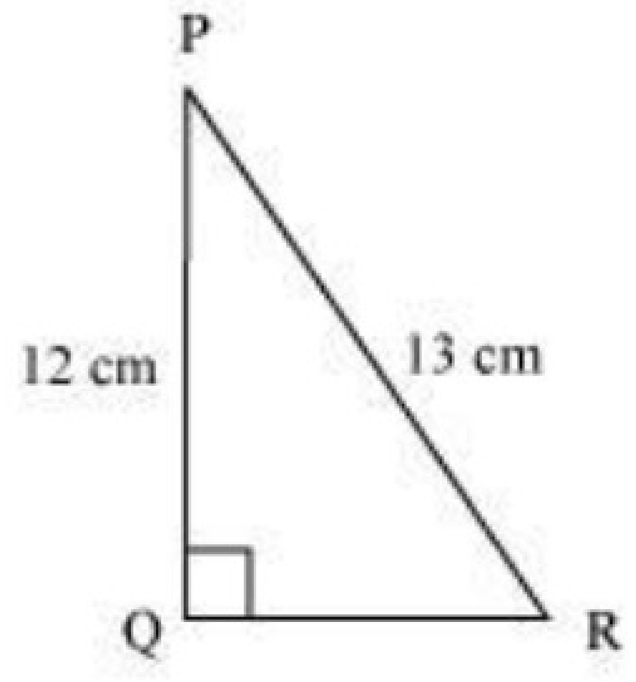
(ii)

$$\sin C = \frac{AB}{AC} = \frac{24}{25}$$

$$\cos C = \frac{BC}{AC} = \frac{7}{25}$$



Question 2: In the given figure find tan P − cot R



Solution: Applying Pythagoras theorem for ΔPQR, we obtain

$PR^2 = PQ^2 + QR^2$

$(13\text{ cm})^2 = (12\text{ cm})^2 + QR^2$

$169\text{ cm}^2 = 144\text{ cm}^2 + QR^2$

$25\text{ cm}^2 = QR^2$

$QR = 5\text{ cm}$

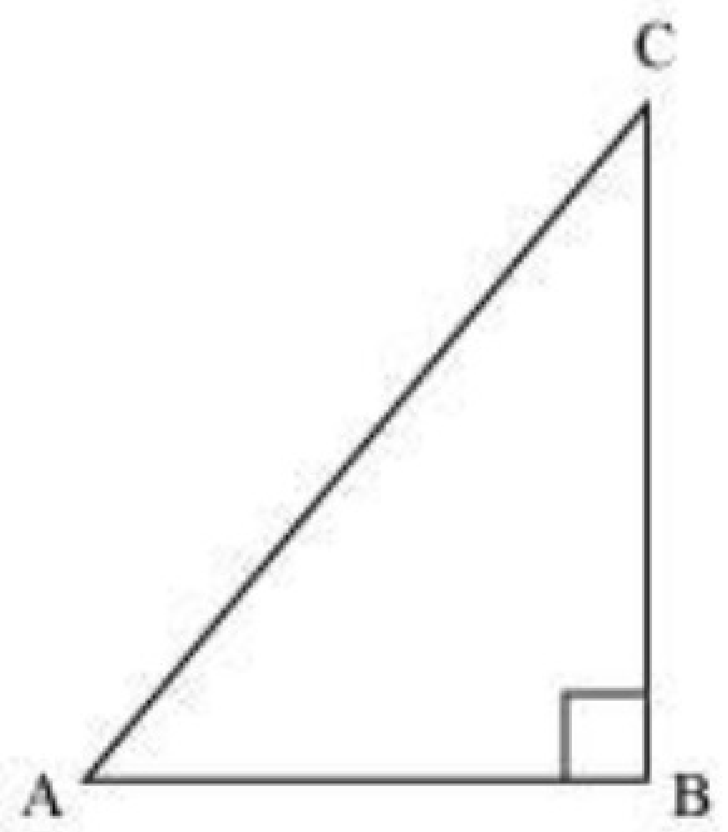
$$\tan P = \frac{QR}{PQ} = \frac{5}{12}$$

$$\cot R = \frac{QR}{PQ} = \frac{5}{12}$$

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

Question 3: If sin A $= \frac{3}{4}$, calculate cos A and tan A.

Solution: Let ΔABC be a right-angled triangle, right-angled at point B.



Given that,

$$\sin A = \frac{3}{4}$$

$$\frac{BC}{AC} = \frac{3}{4}$$

Let BC be 3*k*. Therefore, AC will be 4*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$$AC^2 = AB^2 + BC^2$$

$$(4k)^2 = AB^2 + (3k)^2$$

$$16k^2 - 9k^2 = AB^2$$

$$7k^2 = AB^2$$

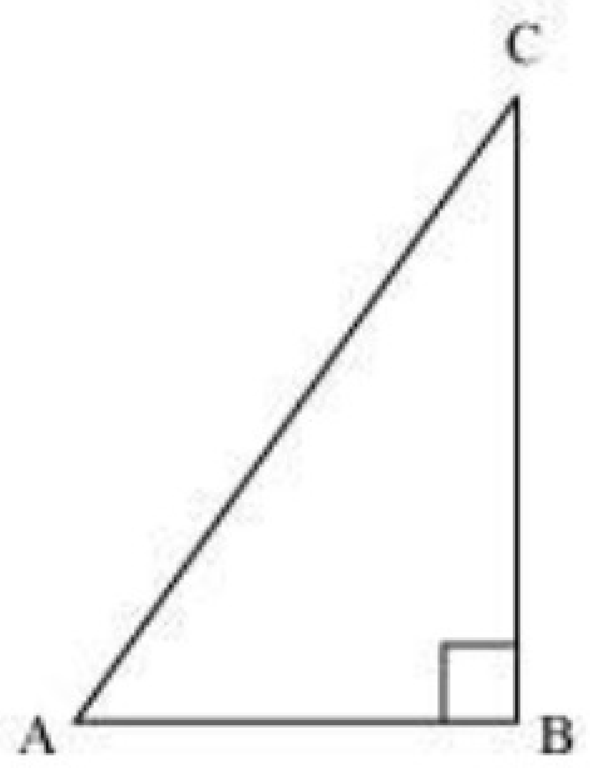
$$AB = \sqrt{7}k$$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

Question 4: Given 15 cot A = 8. Find sin A and sec A

Solution: Consider a right-angled triangle, right-angled at B.



$$\cot A = \frac{AB}{BC}$$

It is given that,

$$\cot A = \frac{AB}{BC} = \frac{8}{15}$$

Let AB be $8k$.Therefore, BC will be $15k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC,

$$AC^2 = AB^2 + BC^2$$

$$= (8k)^2 + (15k)^2$$

$$= 64k^2 + 225k^2$$

$$= 289k^2$$

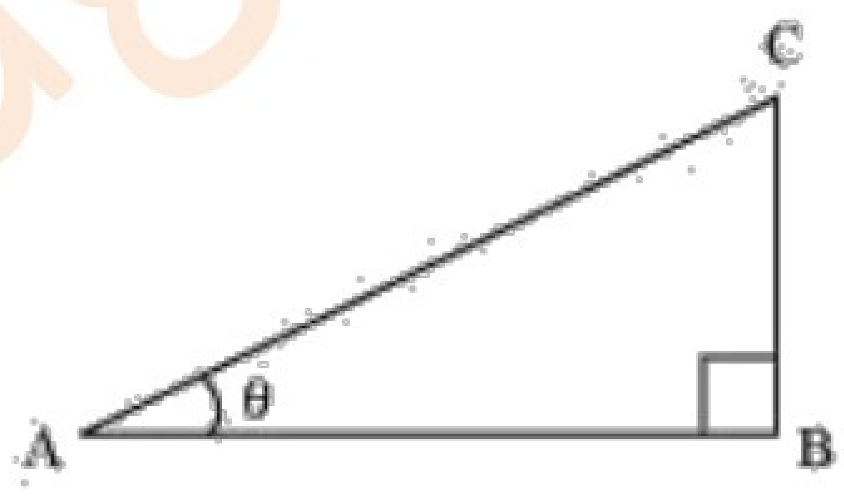
$$AC = 17k$$

$$\sin A = \frac{BC}{AC} = \frac{15}{17}$$

$$\sec A = \frac{AC}{AB} = \frac{17}{8}$$

Question 5: Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Solution: Consider a right-angle triangle ΔABC, right-angled at point B.



$$\sec\theta = \frac{AC}{AB} = \frac{13}{12}$$

If AC is $13k$, AB will be $12k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(13k)^2 = (12k)^2 + (BC)^2$$

$$169k^2 = 144k^2 + BC^2$$

$25k^2 = BC^2$

$BC = 5k$

$$\sin\theta = \frac{13}{12}$$

$$\cos\theta = \frac{12}{13}$$

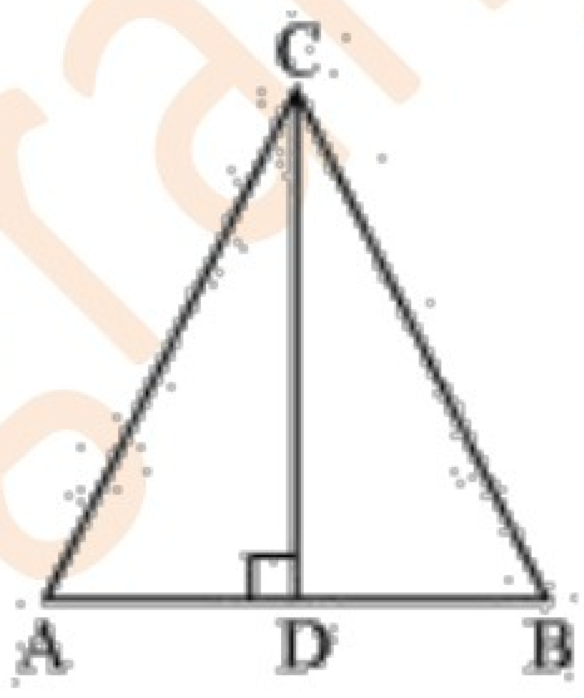
$$\tan\theta = \frac{5}{12}$$

$$\cot\theta = \frac{12}{5}$$

$$\operatorname{cosec}\theta = \frac{13}{5}$$

Question 6: If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

Solution: Let us consider a triangle ABC in which CD ⊥ AB.

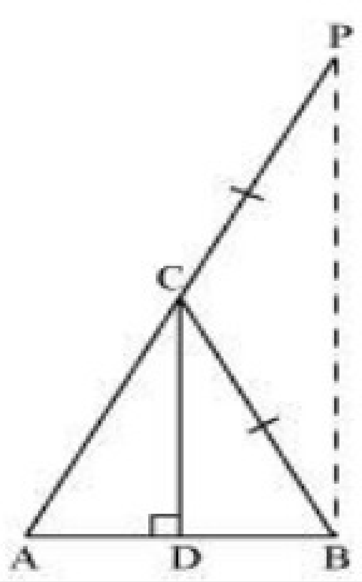


It is given that

cos A = cos B

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC} \quad \ldots (1)$$

We have to prove ∠A = ∠B. To prove this, let us extend AC to P such that BC = CP.



From equation (1), we obtain

$$\frac{AD}{BD} = \frac{AC}{BC}$$
$$\Rightarrow \frac{AD}{BD} = \frac{AC}{CP}$$

By using the converse of B.P.T,

CD || BP

⇒ ∠ACD = ∠CPB (Corresponding angles) … (3)

And, ∠BCD = ∠CBP (Alternate interior angles) … (4)

By construction, we have BC = CP.

∴ ∠CBP = ∠CPB (Angle opposite to equal sides of a triangle) … (5)

From equations (3), (4), and (5), we obtain

∠ACD = ∠BCD … (6)

In ΔCAD and ΔCBD,

∠ACD = ∠BCD [Using equation (6)]

∠CDA = ∠CDB [Both 90°]

Therefore, the remaining angles should be equal.

∴ ∠CAD = ∠CBD

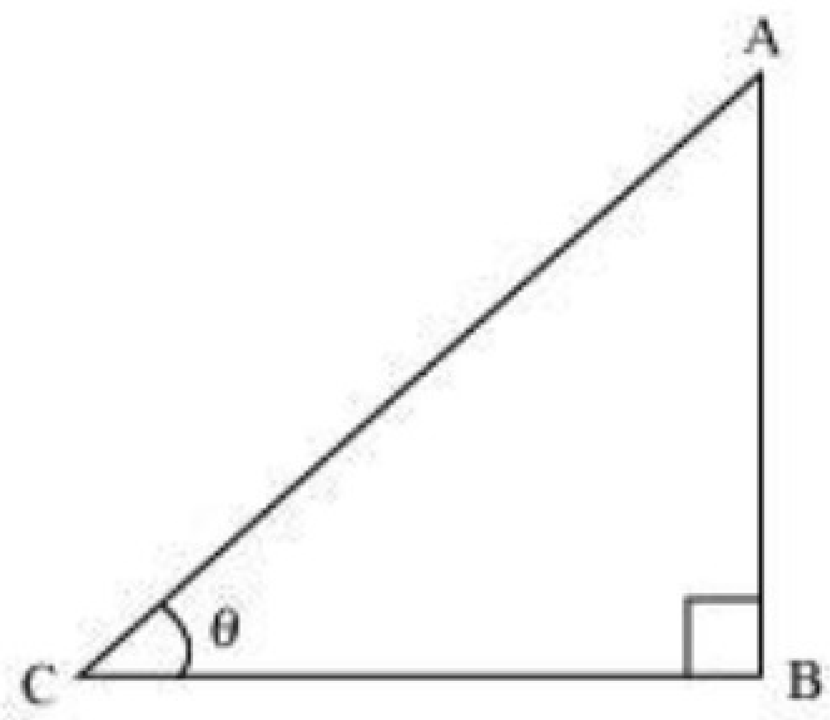
⇒ ∠A = ∠B

Question 7: If $\cot \theta = \frac{7}{8}$, evaluate

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2 \theta$

Solution: Let us consider a right triangle ABC, right-angled at point B.



$$\cot\theta = \frac{7}{8}$$

If BC is $7k$, then AB = $8k$

Applying Pythagoras theorem in ΔABC,

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (7k)^2$

$= 64k^2 + 49k^2$

$= 113k^2$

$$AC = \sqrt{113}k$$

$$\sin\theta = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i)

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = \frac{49}{64}$$

(ii) $\cot^2\theta = (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

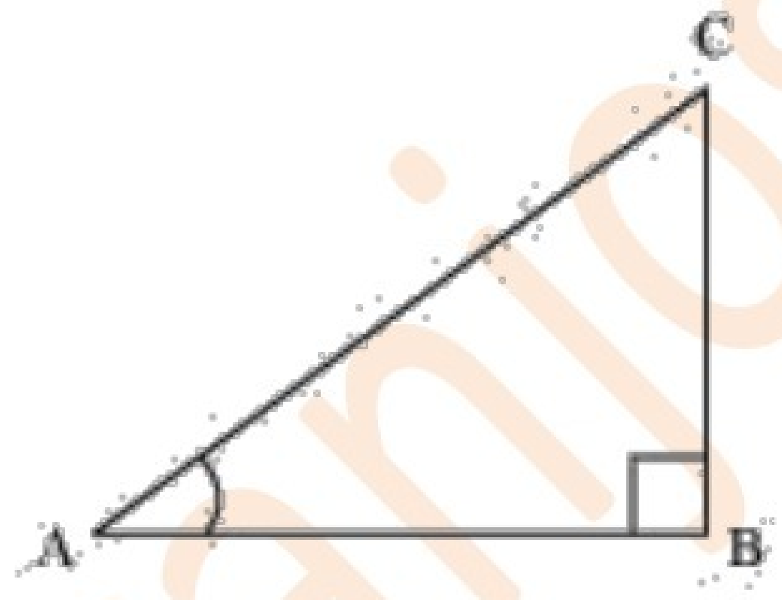
Question 8: If 3 cot A = 4, Check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ *or not*

Solution:

It is given that 3cot A = 4

$$\cot A = \frac{4}{3}$$

Consider a right triangle ABC, right-angled at point B.



$$\cot A = \frac{AB}{BC} = \frac{4}{3}$$

If AB is 4k, then BC will be 3k, where k is a positive integer.

In ΔABC,

$(AC)^2 = (AB)^2 + (BC)^2$

$= (4k)^2 + (3k)^2$

$= 16k^2 + 9k^2$

$= 25k^2$

AC = 5k

$$\sin A = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos A = \frac{4k}{5k} = \frac{4}{5}$$

$$\tan A = \frac{3k}{4k} = \frac{3}{4}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{7}{25}$$

$$\cos^2 A - \sin^2 A$$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{7}{25}$$

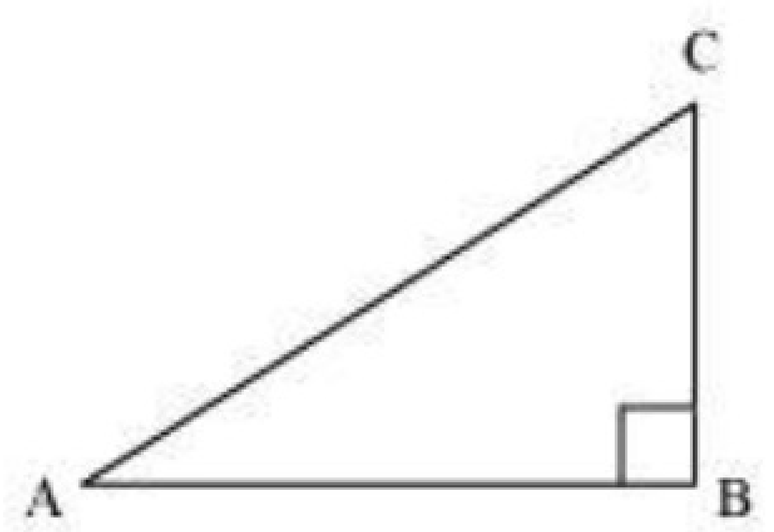
Thus, $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

Question 9: In ΔABC, right angled at B, If $\tan A = \frac{1}{\sqrt{3}}$, find the value of

(i) sin A cos C + cos A sin C

(ii) cos A cos C − sin A sin C

Solution:



$$\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

If BC is k, then AB = $\sqrt{3}k$

In ΔABC,

$$AC^2 = AB^2 + BC^2$$

$= 3k^2 + k^2 = 4k^2$

$\therefore$ AC = 2k

$$\sin A = \frac{k}{2k} = \frac{1}{2}$$
$$\cos A = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$
$$\sin C = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$
$$\cos C = \frac{k}{2k} = \frac{1}{2}$$

(i)

$$\sin A \cos C + \cos A \sin C$$
$$= \frac{1}{2} \times \frac{1}{2} + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{4}{4} = 1$$

(ii)

$$\cos A \cos C - \sin A \sin C$$
$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

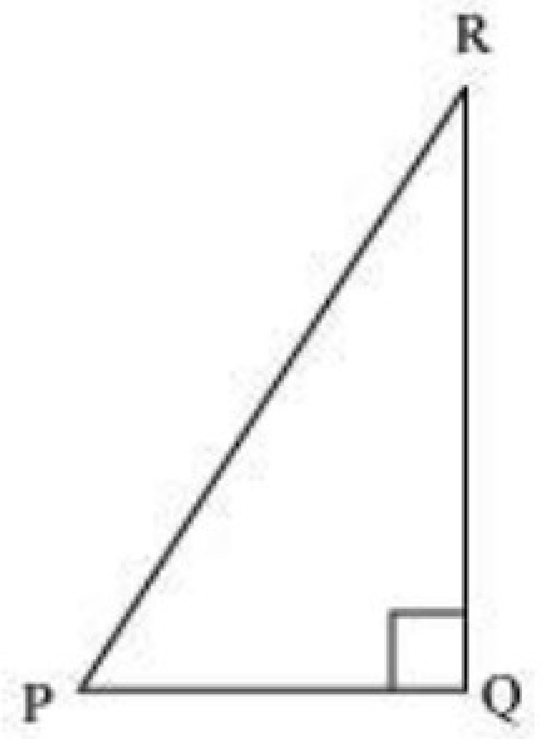
Question 10: In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Solution: Given that, PR + QR = 25

PQ = 5

Let PR be x.

Therefore, QR = 25 − x



Applying Pythagoras theorem in ΔPQR,

$PR^2 = PQ^2 + QR^2$

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

$x = 13$

Therefore, PR = 13 cm

QR = (25 − 13) cm = 12 cm

$\sin P = \frac{12}{13}$

$\cos P = \frac{5}{13}$

$\tan P = \frac{12}{5}$

Question 11: State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) sec A = $\frac{12}{5}$ for some value of angle A.

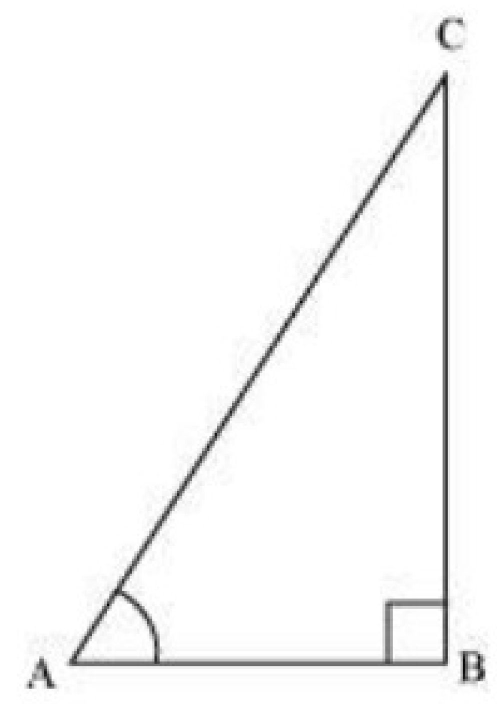
(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A

(v) $\sin \theta = \frac{4}{3}$, for some angle θ

Solution:

(i) Consider a ΔABC, right-angled at B.



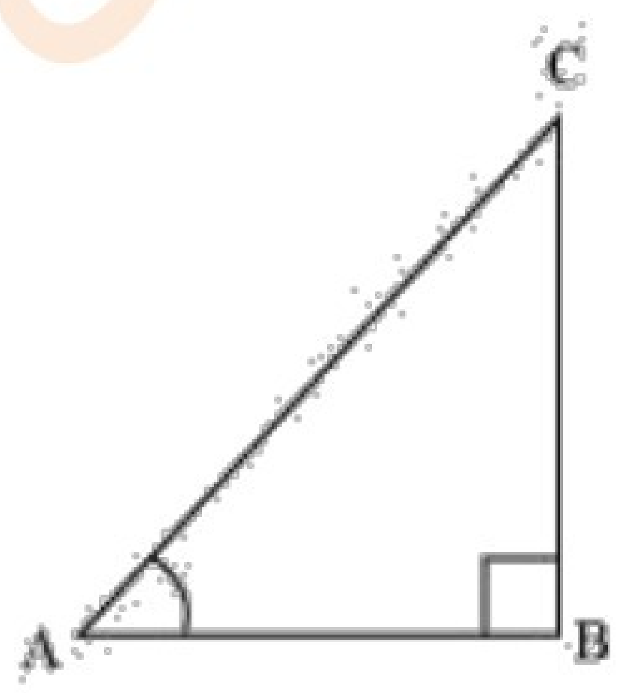
$$\tan A = \frac{12}{5}$$

∴ tan A > 1

So, tan A < 1 is not always true.

Hence, the given statement is false.

(ii) $\sec A = \frac{12}{5}$



$$\frac{AC}{AB} = \frac{12}{5}$$

Let AC be 12*k*, AB will be 5*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$$AC^2 = AB^2 + BC^2$$

$(12k)^2 = (5k)^2 + BC^2$

$144k^2 = 25k^2 + BC^2$

$BC^2 = 119k^2$

$BC = 10.9k$

It can be observed that for given two sides AC = 12k and AB = 5k,

BC should be such that,

$AC - AB < BC < AC + AB$

$12k - 5k < BC < 12k + 5k$

$7k < BC < 17\,k$

However, BC = 10.9k. Clearly, such a triangle is possible and hence, such value of sec A is possible.

Hence, the given statement is true.

(iii) Abbreviation used for cosecant of angle A is cosec A. And cos A is the abbreviation used for cosine of angle A.

Hence, the given statement is false.

(iv) cot A is not the product of cot and A. It is the cotangent of ∠A.

Hence, the given statement is false.

(v) $\sin\theta = \frac{4}{3}$

We know that in a right-angled triangle,

$$\sin\theta = \frac{\text{side opposite to} \angle\theta}{\text{Hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of sin θ is not possible.

Hence, the given statement is false

Exercise: 8.2

Question 1: Evaluate the following

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\dfrac{\cos 45^o}{\sec 30^o + \text{cosec}30^o}$

(iv) $\dfrac{\sin 30^o + \tan 45^o - \text{cosec}60^o}{\sec 30^o + \cos 60^o + \cot 45^o}$

(v) $\dfrac{5\cos^2 60^o + 4\sec^2 30^o - \tan^2 45^o}{\sin^2 30^o + \cos^2 30^o}$

Solution:

(i) $\sin60° \cos30° + \sin30° \cos 60°$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{3}{4} + \frac{1}{4}$$
$$= 1$$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$$= 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$
$$= 2$$

(iii)

$$\frac{\cos 45^\circ}{\sec 30^\circ + \text{cosec}30^\circ}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}\left(2+2\sqrt{3}\right)} = \frac{\sqrt{3}}{2\left(\sqrt{2}+\sqrt{6}\right)}$$

$$= \frac{\sqrt{3}}{2\left(\sqrt{2}+\sqrt{6}\right)} \cdot \frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}}$$

$$= \frac{\sqrt{6}-3\sqrt{2}}{2\left(2-6\right)}$$

$$= \frac{\sqrt{6}-3\sqrt{2}}{-8}$$

$$= \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv)

$$\frac{\sin 30^\circ + \tan 45^\circ - \cos ec 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$= \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{3}{2}-\frac{2}{\sqrt{3}}}{\frac{3}{2}+\frac{2}{\sqrt{3}}}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4}$$

$$= \frac{27+16-24\sqrt{3}}{27-16}$$

$$= \frac{43-24\sqrt{3}}{11}$$

(v)

$$\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - 1^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{1}{4} + \frac{16}{3}}{1}$$

$$= \frac{1}{4} + \frac{16}{3}$$

$$= \frac{67}{12}$$

Question 2: Choose the correct option and justify your choice.

(i) $\frac{2\tan 30^o}{1+\tan^2 30^o} =$

(A) sin60° (B) cos60° (C) tan60° (D) sin30°

(ii) $\frac{1-\tan^2 45^o}{1+\tan^2 45^o} =$

(A) tan90° (B) 1 (C) sin45° (D) 0

(iii) sin2A = 2sinA is true when A =

(A) 0° (B) 30° (C) 45° (D) 60°

(iv) $\frac{2\tan 30^o}{1-\tan^2 30^o}$ =

(A) cos60° (B) sin60° (C) tan60° (D) sin30°

Solution:

(i)

$$\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$$

$$=\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$=\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}$$

$$=\frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}=\frac{6}{4\sqrt{3}}$$

$$=\frac{\sqrt{3}}{2}=\sin 60^\circ$$

Hence, (A) is correct.

(ii)

$$\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}=\frac{1-1^2}{1+1^2}=\frac{1-1}{1+1}=0$$

Hence, (D) is correct.

(iii) Out of the given alternatives, only A = 0° is correct.

As $\sin 2A = \sin 0° = 0$

$2 \sin A = 2\sin 0° = 2(0) = 0$

Hence, (A) is correct.

(iv)

$$\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$$

$$= \tan 60^\circ$$

Hence, (C) is correct.

Question 3: If $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$; $0° < A + B \leq 90°$. $A > B$ find A and B.

Solution:

$$\tan(A+B)=\sqrt{3}$$
$$\tan(A+B)=\tan 60^\circ$$

$\Rightarrow$ $A + B = 60$ … (1)

$$\tan(A-B)=\frac{1}{\sqrt{3}}$$

$\Rightarrow$ $\tan(A - B) = \tan 30$

$\Rightarrow \quad A - B = 30 \ldots (2)$

On adding both equations, we obtain

$2A = 90$

$\Rightarrow \quad A = 45$

From equation (1), we obtain

$45 + B = 60$

$B = 15$

Therefore, $\angle A = 45°$ and $\angle B = 15°$

Question 4: State whether the following are true or false. Justify your answer.

(i) $\sin (A + B) = \sin A + \sin B$
(ii) The value of $\sin\theta$ increases as θ increases
(iii) The value of $\cos \theta$ increases as θ increases
(iv) $\sin\theta = \cos \theta$ for all values of θ
(v) $\cot A$ is not defined for $A = 0°$

Solution:

(i) $\sin (A + B) = \sin A + \sin B$

Let $A = 30°$ and $B = 60°$

$\sin (A + B) = \sin (30° + 60°)$

$= \sin 90°$

$= 1$

$\sin A + \sin B = \sin 30° + \sin 60°$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{1+\sqrt{3}}{2}$$

Clearly, $\sin (A + B) \neq \sin A + \sin B$

Hence, the given statement is false.

(ii) The value of sin θ increases as θ increases in the interval of 0° < θ < 90° as

$$\sin 0^\circ = 0$$

$$\sin 30^\circ = \frac{1}{2} = 0.5$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\sin 90^\circ = 1$$

Hence, the given statement is true.

(iii) $\cos 0^\circ = 1$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos 60^\circ = \frac{1}{2} = 0.5$$

$$\text{Cos } 90^\circ = 0$$

It can be observed that the value of cos θ does not increase in the interval of 0° < θ < 90°.

Hence, the given statement is false.

(iv) sin θ = cos θ for all values of θ.

This is true when θ = 45°

As $\sin 45^\circ = \frac{1}{\sqrt{2}}$ and $\cos 45^\circ = \frac{1}{\sqrt{2}}$

It is not true for all other values of θ. Hence, the given statement is false.

(v) cot A is not defined for A = 0°

$$\cot A = \frac{\cos A}{\sin A}$$

$$\cot 0^\circ = \frac{\cos 0^\circ}{\sin 0^\circ} = \frac{1}{0} = \infty$$

Hence, the given statement is true.

Exercise: 8.3

Question 1: Evaluate:

(I) $\frac{\sin 18^o}{\cos 72^o}$

(II) $\frac{\tan 26^o}{\cot 64^o}$

(III) cos 48° − sin 42°

(IV)cosec 31° − sec 59°

Solution:

(I) $\frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\sin\left(90^\circ - 72^\circ\right)}{\cos 72^\circ} = \frac{\cos 72^\circ}{\cos 72^\circ} = 1$

(II) $\frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\tan\left(90^\circ - 64^\circ\right)}{\cot 64^\circ} = \frac{\cot 64^\circ}{\cot 64^\circ} = 1$

(III) cos 48° − sin 42° = cos (90°− 42°) − sin 42°

= sin 42° − sin 42°

= 0

(IV) cosec 31° − sec 59° = cosec (90° − 59°) − sec 59°

= sec 59° − sec 59°

= 0

Question 2: Show that

(I) tan 48° tan 23° tan 42° tan 67° = 1

(II) cos 38° cos 52° − sin 38° sin 52° = 0

Solution:

(I) tan 48° tan 23° tan 42° tan 67°

= tan (90° − 42°) tan (90° − 67°) tan 42° tan 67°

= cot 42° cot 67° tan 42° tan 67°

= (cot 42° tan 42°) (cot 67° tan 67°)

= (1) (1)

= 1

(II) cos 38° cos 52° − sin 38° sin 52°

= cos (90° − 52°) cos (90°−38°) − sin 38° sin 52°

= sin 52° sin 38° − sin 38° sin 52°

= 0

Question 3: If tan 2A = cot (A− 18°), where 2A is an acute angle, find the value of A.

Solution:

Given that,

tan 2A = cot (A− 18°)

cot (90° − 2A) = cot (A −18°)

$90° - 2A = A - 18°$

$108° = 3A$

$A = 36°$

Question 4: If tan A = cot B, prove that A + B = 90°

Solution:

Given that,

$\tan A = \cot B$

$\tan A = \tan (90° - B)$

$A = 90° - B$

$A + B = 90°$

Question 5: If sec 4A = cosec (A− 20°), where 4A is an acute angle, find the value of A.

Solution:

Given that,

$\sec 4A = \text{cosec} (A - 20°)$

$\text{cosec} (90° - 4A) = \text{cosec} (A - 20°)$

$90° - 4A = A - 20°$

$110° = 5A$

$A = 22°$

Question 6: If A, Band C are interior angles of a triangle ABC then show that

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

Solution: We know that for a triangle ABC,

$\angle A + \angle B + \angle C = 180°$

$\angle B + \angle C = 180° - \angle A$

$$\frac{\angle B + \angle C}{2} = 90^\circ - \frac{\angle A}{2}$$
$$\sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$$
$$= \cos\frac{A}{2}$$

Question 7: Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Solution:

$\sin 67° + \cos 75°$

$= \sin (90° - 23°) + \cos (90° - 15°)$

$= \cos 23° + \sin 15°$

Exercise: 8.4

Question 1: Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Solution:

We know that,

$$\text{cosec}^2 A = 1 + \cot^2 A$$
$$\frac{1}{\text{cosec}^2 A} = \frac{1}{1+\cot^2 A}$$
$$\sin^2 A = \frac{1}{1+\cot^2 A}$$
$$\sin A = \pm\frac{1}{\sqrt{1+\cot^2 A}}$$

$\sqrt{1+\cot^2 A}$ will always be positive as we are adding two positive quantities.

Therefore, $\sin A = \dfrac{1}{\sqrt{1+\cot^2 A}}$

Also,

$$\sec^2 A = 1 + \tan^2 A$$

$$\sec^2 A = 1 + \frac{1}{\cot^2 A}$$

$$\sec^2 A = \frac{1+\cot^2 A}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{1+\cot^2 A}}{\cot A}$$

Question 2: Write all the other trigonometric ratios of ∠A in terms of sec A.

Solution:

We know that,

$$\sin^2 A + \cos^2 A = 1$$

$$\sin^2 A = 1 - \cos^2 A$$

$$\sin A = \sqrt{1-\left(\frac{1}{\sec A}\right)^2}$$

$$\sin A = \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}$$

$$\sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\tan^2 A + 1 = \sec^2 A$$

$$\tan^2 A = \sec^2 A - 1$$

$$\tan A = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{\cos A}{\sin A}$$

$$\cot A = \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$\cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\cos ec A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

Question 3: Evaluate

(i) $\dfrac{\sin^2 63^o + \sin^2 27^o}{\cos^2 17^o + \cos^2 73^o}$

(ii) sin25° cos65° + cos25° sin65°

Solution:

(i)

$$\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$= \frac{\sin^2 (90-27)^\circ + \sin^2 27^\circ}{\cos^2 (90-73)^\circ + \cos^2 73^\circ}$$

$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$$

$$= \frac{1}{1} = 1$$

(As $\sin^2 A + \cos^2 A = 1$)

(ii) sin25° cos65° + cos25° sin65°

$$\sin 25^\circ \left\{\cos\left(90^\circ - 25^\circ\right)\right\} + \cos 25^\circ \left\{\sin\left(90^\circ - 25^\circ\right)\right\}$$

$$= \sin 25^\circ \sin 25^\circ + \cos 25^\circ \cos 25^\circ$$

$= \sin^2 25° + \cos^2 25°$

$= 1$ (As $\sin^2 A + \cos^2 A = 1$)

Question 4: Choose the correct option. Justify your choice.

(i) $9\sec^2 A - 9\tan^2 A =$

(A) 1 (B) 9 (C) 8 (D) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

(A) 0 (B) 1 (C) 2 (D) −1

(iii) $(\sec A + \tan A)(1 - \sin A) =$

(A) secA (B) sinA (C) cosecA (D) cosA

(iv) $\dfrac{1+\tan^2 A}{1+\cot^2 A}$

(A) $\sec^2 A$ (B) −1 (C) $\cot^2 A$ (D) $\tan^2 A$

Solution:

(i) $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9(1)$ [As $\sec^2 A - \tan^2 A = 1$]

$= 9$

Hence, alternative (B) is correct.

(ii)

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$$\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$=\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

$$=\left[\frac{(\sin\theta+\cos\theta)^2-1^2}{\sin\theta\cos\theta}\right]$$

$$=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$
$$= 2$$

Hence, alternative (C) is correct.

(iii) (secA + tanA) (1 − sinA)

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$
$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A}$$
$$= \frac{\cos^2 A}{\cos A}$$
$$= \cos A$$

Hence, alternative (D) is correct.

(iv)

$$\frac{1 + \tan^2 A}{1 + \cot^2 A}$$
$$= \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}} = \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}}$$
$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A}$$
$$= \tan^2 A$$

Hence, alternative (D) is correct.

Question 5: Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

Solution:

(i)

$$(\cos ec\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$$

LHS

$$(\cos ec\theta - \cot\theta)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

$$= RHS$$

(ii)

$$\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$$

LHS

$$\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{1+1+2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{2(1+\sin A)}{\cos A(1+\sin A)}$$

$$= 2\sec A$$

$$= RHS$$

(iii)

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \cos ec\theta$$

LHS

$$= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \cos ec\theta \sec\theta + 1$$

= R.H.S.

(iv)

$$\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$$

LHS

$$\frac{1+\sec A}{\sec A}$$

$$=\frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$=\frac{\frac{1+\cos A}{\cos A}}{\frac{1}{\cos A}}$$

$$=1+\cos A$$

$$=\frac{(1+\cos A)(1-\cos A)}{(1-\cos A)}$$

$$=\frac{1-\cos^2 A}{1-\cos A}$$

$$=\frac{\sin^2 A}{1-\cos A}$$

= R.H.S

(v)

$$\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\cos ecA+\cot A$$

LHS

$$=\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$$

Divide the equation by sinA

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

Using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$,

$$= \frac{\cot A - (1 - \operatorname{cosec} A)}{\cot A + (1 - \operatorname{cosec} A)} \times \frac{\cot A - (1 - \operatorname{cosec} A)}{\cot A - (1 - \operatorname{cosec} A)}$$

$$= \frac{\cot^2 A + (1 - \operatorname{cosec} A)^2 - 2\cot A(1 - \operatorname{cosec} A)}{\cot^2 A - (1 - \operatorname{cosec} A)^2}$$

$$= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A - 2\cot A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \frac{\operatorname{cosec}^2 A + \operatorname{cosec}^2 A - 2\operatorname{cosec} A - 2\cot A + 2\cot A \operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$$

$$= \frac{2\operatorname{cosec}^2 A - 2\operatorname{cosec} A - 2\cot A + 2\cot A \operatorname{cosec} A}{-1 - 1 + 2\operatorname{cosec} A}$$

$$= \frac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{-2 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{2\operatorname{cosec} A - 2}$$

$$= \operatorname{cosec} A + \cot A$$

= R.H.S

(vi)

$$\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

LHS

$$= \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \sec A + \tan A$$

RHS

(vii)

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$$

LHS

$$= \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta\left[2(1-\sin^2\theta)-1\right]}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2-2\sin^2\theta-1)}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(1-2\sin^2\theta)}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

$$= RHS$$

(viii)

$$(\sin A+\cos ecA)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$$

LHS

$$(\sin A+\cos ecA)^2+(\cos A+\sec A)^2$$

$$\sin^2 A+\cos ec^2 A+2\sin A\cos ecA+\cos^2+\sec^2 A+2\cos A\sec A$$

$$=\sin^2 A+\cos ec^2 A+2+\cos^2 A+\sec^2 A+2$$

$$=\sin^2 A+\cos^2 A+4+\cos ec^2 A+\sec^2 A$$

$$=1+4+1+\cot^2 A+1+\tan^2 A$$

$$=7+\tan^2 A+\cot^2 A$$

$$=RHS$$

(ix)

$$(\cos ecA-\sin A)(\sec A-\cos A)=\frac{1}{\tan A+\cot A}$$

LHS

$$(\cos ecA-\sin A)(\sec A-\cos A)$$

$$=\left(\frac{1}{\sin A}-\sin A\right)\left(\frac{1}{\cos A}-\cos A\right)$$

$$=\left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$$

$$=\left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)$$

$$=\cos A\sin A$$

RHS

$$=\frac{1}{\tan A+\cot A}$$

$$=\frac{1}{\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}}$$

$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$
$$= \sin A \cos A$$

Hence, L.H.S = R.H.S

(x)

$$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$$

$$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)$$
$$= \frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}}$$
$$= \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\cos^2 A + \sin^2 A}{\sin^2 A}}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$
$$= \frac{\sin^2 A}{\cos^2 A}$$
$$= \tan^2 A$$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2$$
$$= \left(\frac{1-\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}\right)^2$$

$$= \left(\frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}} \right)^2$$

$$= \left(\frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\cos A - \sin A}{-\sin A}} \right)^2$$

$$= \left(\frac{\frac{1}{\cos A}}{\frac{1}{-\sin A}} \right)^2$$

$$= \left(\frac{-\sin A}{\cos A} \right)^2$$

$$= \tan^2 A$$