Chapter 4
Quadratic Equations

Exercise: 4.1

Question 1: Check whether the following are quadratic equations:

(i) $(x+1)^2 = 2(x-3)$

(ii) $x^2 - 2x = (-2)(3-x)$

(iii) $(x-2)(x+1) = (x-1)(x+3)$

(iv) $(x-3)(2x+1) = x(x+5)$

(v) $(2x-1)(x-3) = (x+5)(x-1)$

(vi) $x^2 + 3x + 1 = (x-2)^2$

(vii) $(x+2)^3 = 2x(x^2-1)$

(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

Solution:

(i) $(x+1)^2 = 2(x-3)$

$$x^2 + 1 + 2x = 2x - 6$$

$$x^2 + 7 = 0$$

It is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3-x)$

$$x^2 - 2x = -6 + 2x$$

$$x^2 - 4x + 6 = 0$$

It is a quadratic equation.

(iii) $(x-2)(x+1) = (x-1)(x+3)$

$$x^2 - x - 2 = x^2 + 2x - 3$$
$$-3x + 1 = 0$$

It is not a quadratic equation.

(iv) $(x-3)(2x+1) = x(x+5)$

$$2x^2 - 5x - 3 = x^2 + 5x$$
$$x^2 - 10x - 3 = 0$$

It is a quadratic equation.

(v) $(2x-1)(x-3) = (x+5)(x-1)$

$$2x^2 - 6x - x + 3 = x^2 + 4x - 5$$
$$x^2 - 11x - 2 = 0$$

It is a quadratic equation.

(vi) $x^2 + 3x + 1 = (x-2)^2$

$$x^2 + 3x + 1 = x^2 + 4 - 4x$$
$$7x - 3 = 0$$

It is not a quadratic equation.

(vii) $(x+2)^3 = 2x(x^2-1)$

$$x^3 + 8 + 6x(x+2) = 2x^3 - 2x$$
$$-x^3 + 8 + 6x^2 + 12x + 2x = 0$$
$$-x^3 + 6x^2 + 14x + 8 = 0$$

It is not a quadratic equation.

(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

$$x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x(x-2)$$
$$x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$$

$2x^2 - 13x - 9 = 0$ It is a quadratic equation.

Question 2: Represent the following situations in the form of quadratic equations.

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Solution:

(i)

Let the breadth $= x\ m$

Length $= (2x+1)m$

Area $= l \times b$

$528 = x(2x+1)$

$2x^2 + x - 528$

(ii)

Let the consecutive integers are x and $(x+1)$

Now,

$$x(x+1) = 306$$

$$x^2 + x - 306 = 0$$

(iii)

Let Rohan's age $= x$

mother's age $= x + 26$

Now,

$$(x+3)(x+29) = 360$$

$$x^2 + 32x - 273 = 0$$

(iv) Let the speed of train = x km/h

Distance = 480 km

Time taken $= \frac{480}{x} hrs$

Now,

$$\left(x-8\right)\left(\frac{480}{x}+3\right)=480$$
$$\left(x-8\right)\left(480+3x\right)=480x$$
$$480x+3x^2-3840-24x-480x=0$$
$$3x^2-24x-3840=0$$
$$x^2-8x-1280=0$$

Exercise: 4.2

Question 1: Find the roots of the following quadratic equations by factorisation:

(i) $x^2-3x-10=0$

(ii) $2x^2+x-6=0$

(iii) $\sqrt{2}x^2+7x+5\sqrt{2}=0$

(iv) $2x^2-x+\frac{1}{8}=0$

(v) $100x^2-20x+1=0$

Solution:

(i)
$$x^2-3x-10=0$$
$$x^2-5x+2x-10=0$$
$$x(x-5)+2(x-5)=0$$
$$\left(x-5\right)\left(x+2\right)=0$$
$$x-5=0, \qquad x+2=0$$
$$x=5, \qquad x=-2$$

Roots are $-2,5$

(ii)

$$2x^2 + x - 6 = 0$$
$$2x^2 + 4x - 3x - 6 = 0$$
$$2x(x+2) - 3(x+2) = 0$$
$$(2x-3)(x+2) = 0$$
$$x = -2, \frac{3}{2}$$

Roots are $-2, \frac{3}{2}$

(iii)

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$
$$\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$$
$$x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$$
$$(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$$
$$x = -\frac{5}{\sqrt{2}}, -\sqrt{2}$$

Roots are $-\frac{5}{\sqrt{2}}, -\sqrt{2}$

(iv)

$$2x^2 - x + \frac{1}{8} = 0$$
$$16x^2 - 8x + 1 = 0$$
$$16x^2 - 4x - 4x + 1 = 0$$
$$4x(4x-1) - 1(4x-1) = 0$$
$$(4x-1)^2 = 0$$
$$x = \frac{1}{4}$$

Roots are $\frac{1}{4}, \frac{1}{4}$

(v)

$$100x^2 - 20x + 1 = 0$$
$$100x^2 - 10x - 10x + 1 = 0$$
$$10x(10x - 1) - 1(10x - 1) = 0$$
$$(10x - 1)^2 = 0$$
$$x = \frac{1}{10}$$

Roots are $\frac{1}{10}, \frac{1}{10}$

Question 2:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. Find out the number of toys produced on that day.

Solutions:

(i) Let the number of John's marbles = x.

Therefore, number of Jivanti's marble = $45 - x$

Product of their marbles is 124.

Now,

$$(x - 5)(40 - x) = 124$$
$$x^2 - 45x + 324 = 0$$
$$x^2 - 36x - 9x + 324 = 0$$
$$x(x - 36) - 9(x - 36) = 0$$
$$(x - 36)(x - 9) = 0$$
$$x = 9, 36$$

If the number of John's marbles = 36,

Then, number of Jivanti's marbles = 45 − 36 = 9

If number of John's marbles = 9,

Then, number of Jivanti's marbles = 45 − 9 = 36

(ii)

Let the number of toys produced be x.

∴ Cost of production of each toy = Rs (55 − x)

Total production of the toys = Rs 750

$$x(55-x)=750$$
$$x^2-55x+750=0$$
$$x^2-25x-30x+750=0$$
$$x(x-25)-30(x-25)=0$$
$$(x-25)(x-30)=0$$
$$x=25,30$$

Hence, the number of toys will be either 25 or 30.

Question 3: Find two numbers whose sum is 27 and product is 182.

Solution:

Let the first number be x and the second number is 27 − x.

Therefore, their product = x (27 − x)

It is given that the product of these numbers is 182.

$$x\left(27-x\right)=182$$
$$x^2-27x+182=0$$
$$x^2-13x-14x+182=0$$
$$x(x-13)-14(x-13)=0$$
$$(x-13)(x-14)=0$$
$$x=13,14$$

If first number = 13, then

Other number = 27 − 13 = 14

If first number = 14, then

Other number = 27 − 14 = 13

Therefore, the numbers are 13 and 14.

Question 4: Find two consecutive positive integers, sum of whose squares is 365.

Solution:

Let the consecutive positive integers be x and $x + 1$.

$$
\begin{aligned}
&x^2 + (x+1)^2 = 365 \\
&x^2 + x^2 + 1 + 2x - 365 = 0 \\
&2x^2 + 2x - 364 = 0 \\
&x^2 + x - 182 = 0 \\
&x^2 + 14x - 13x - 182 = 0 \\
&x(x+14) - 13(x+14) = 0 \\
&(x+14)(x-13) = 0 \\
&x = 13, -14
\end{aligned}
$$

Since the integers are positive, x can only be 13.

∴ $x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers will be 13 and 14.

Question 5: The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Solution:

Let the base of the right triangle be x cm.

Its altitude = $(x - 7)$ cm

Pythagoras theorem

$$x^2 + (x-7)^2 = 13^2$$
$$x^2 + x^2 + 49 - 14x - 169 = 0$$
$$2x^2 - 14x - 120 = 0$$
$$x^2 - 7x - 60 = 0$$
$$x^2 - 12x + 5x - 60 = 0$$
$$x(x-12) + 5(x-12) = 0$$
$$(x-12)(x+5) = 0$$
$$x = 12, -5$$

Since sides are positive, x can only be 12.

Therefore, the base of the given triangle is 12 cm and the altitude of this triangle will be (12 − 7) cm = 5 cm.

Question 6: A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

Solution:

Let the number of articles produced be x.

Therefore, cost of production of each article = Rs $(2x + 3)$

Total production is Rs 90.

$$x(2x+3) = 90$$
$$2x^2 + 3x - 90 = 0$$
$$2x^2 + 15x - 12x - 90 = 0$$
$$x(2x+15) - 6(2x+15) = 0$$
$$(2x+15)(x-6) = 0$$
$$x = 6, \frac{-15}{2}$$

As the number of articles produced can only be a positive integer, therefore, x can only be 6.

Hence, number of articles produced = 6

Cost of each article = 2 × 6 + 3 = Rs 15

Exercise: 4.3

Question 1: Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + \sqrt{3} = 0$

(iv) $2x^2 + x + 4 = 0$

Solution:

(i) $2x^2 - 7x + 3 = 0$

$2x^2 - 7x = -3$

Divide the equation by 2

$$x^2 - \frac{7}{2}x = -\frac{3}{2}$$

$$x^2 - 2 \times x \times \frac{7}{4} = -\frac{3}{2}$$

$$x^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 = -\frac{3}{2} + \left(\frac{7}{4}\right)^2$$

$$\left(x - \frac{7}{4}\right)^2 = \frac{9}{16} - \frac{3}{2}$$

$$\left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$$

$$\left(x - \frac{7}{4}\right) = \pm\frac{5}{4}$$

$$x - \frac{7}{4} = \frac{5}{4}, \qquad x - \frac{7}{4} = -\frac{5}{4}$$

$$x = 3, \qquad x = \frac{1}{2}$$

(ii) $2x^2 + x - 4 = 0$

$$x^2 + \frac{x}{2} = 2$$

$$x^2 + 2.x.\frac{1}{4} = 2$$

$$x^2 + 2.x.\frac{1}{4} + \left(\frac{1}{4}\right)^2 = 2 + \left(\frac{1}{4}\right)^2$$

$$\left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$$

$$x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$$

$$x + \frac{1}{4} = \frac{\sqrt{33}}{4}, \qquad x + \frac{1}{4} = -\frac{\sqrt{33}}{4}$$

$$x = \frac{\sqrt{33} - 1}{4}, \qquad x = \frac{-\sqrt{33} - 1}{4}$$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$$(2x)^2 + 2 \times 2x \times \sqrt{3} + \left(\sqrt{3}\right)^2 = 0$$

$$\left(2x + \sqrt{3}\right)^2 = 0$$

$$2x + \sqrt{3} = 0$$

$$x = -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

(iv) $2x^2 + x + 4 = 0$

$$x^2 + \frac{x}{2} = -2$$

$$x^2 + 2 \times x \times \frac{1}{4} = -2$$

$$x^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = -2 + \left(\frac{1}{4}\right)^2$$

$$\left(x+\frac{1}{4}\right)^2=\frac{-31}{16}$$

Square of a number can not be negative

Question 2: Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

Solution:

(i)

$$2x^2-7x+3=0$$

Compare the equation with $ax^2-bx+c=0$.

We got,

$$a=2,\ b=-7,\ c=3$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{7\pm\sqrt{49-24}}{4}$$

$$x=\frac{7\pm\sqrt{25}}{4}$$

$$x=\frac{7\pm5}{4}$$

$$x=3,\frac{1}{2}$$

(ii)

$$2x^2+x-4=0$$

Compare the equation with $ax^2-bx+c=0$.

We got,

$$a=2,\ b=1,\ c=-4$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1+32}}{4}$$
$$x = \frac{-1 \pm \sqrt{33}}{4}$$
$$x = \frac{-1+\sqrt{33}}{4}, \frac{-1-\sqrt{33}}{4}$$

(iii)

$$4x^2 + 4\sqrt{3}x + 3 = 0$$

Compare the equation with $ax^2 - bx + c = 0$.

We got,

$$a = 4,\ b = 4\sqrt{3},\ c = 3$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4\sqrt{3} \pm \sqrt{48-48}}{8}$$
$$x = \frac{-4\sqrt{3} \pm 0}{8}$$
$$x = \frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$$

(iv)

$$2x^2 + x + 4 = 0$$

Compare the equation with $ax^2 - bx + c = 0$.

We got,

$$a = 2,\ b = 1,\ c = 4$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-1 \pm \sqrt{1-32}}{4}$$

$$x = \frac{-1 \pm \sqrt{-31}}{4}$$

Square of a number can not be negative.

Question 3: Find the roots of the following equations:

(i) $x - \frac{1}{x} = 3,\ x \neq 0$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30},\ x \neq -4, 7$

Solution:

(i)

$$x - \frac{1}{x} = 3$$
$$x^2 - 3x - 1 = 0$$

Compare the equation with $ax^2 - bx + c = 0$.

We got,

$$a = 1,\ b = -3,\ c = -1$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{3 \pm \sqrt{9+4}}{2}$$
$$x = \frac{3 \pm \sqrt{13}}{2}$$
$$x = \frac{3+\sqrt{13}}{2}, \frac{3-\sqrt{13}}{2}$$

(ii)

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$
$$30(x-7-x-4) = 11(x^2 - 3x - 28)$$
$$30(-11) = 11(x^2 - 3x - 28)$$

$$x^2 - 3x - 28 = -30$$
$$x^2 - 3x + 2 = 0$$
$$x^2 - 2x - x + 2 = 0$$
$$x(x-2) - 1(x-2) = 0$$
$$(x-2)(x-1) = 0$$
$$x = 1, 2$$

Question 4: The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is 1/3. Find his present age.

Solution:

Let the present age of Rehman $= x$ yrs

3 yrs ago, his age was $= (x-3)\ yrs$

5 yrs hance, his age will be $= (x+5)\ yrs$

Now,

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$
$$3(x+5+x-3) = (x-3)(x+5)$$
$$3(2x+2) = x^2 + 2x - 15$$
$$6x + 6 = x^2 + 2x - 15$$
$$x^2 - 4x - 21 = 0$$
$$x^2 - 7x + 3x - 21 = 0$$
$$x(x-7) + 3(x-7) = 0$$
$$(x-7)(x+3) = 0$$
$$x = 7, -3$$

Age cannot be negative. Thus, Rehman's age = 7 yrs

Question 5: In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Solution:

Let the marks in maths = x

Marks in english $= 30 - x$

Now,

$$(x+2)(30-x-3) = 210$$
$$(x+2)(27-x) = 210$$
$$27x - x^2 + 54 - 2x = 210$$
$$x^2 - 25x + 156 = 0$$
$$x^2 - 12x - 13x + 156 = 0$$
$$x(x-12) - 13(x-12) = 0$$
$$(x-12)(x-13) = 0$$
$$x = 12, 13$$

If marks in maths are 12, then marks in english will be 18.

If marks in maths are 13, then marks in english will be 17.

Question 6: The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Solution:

Let the shorter side of a rectangle $= x$ m

The longer side of a rectangle $= (x+30)$ m

Diagonal $= \sqrt{x^2 + (x+30)^2}$

Now,

$$\sqrt{x^2 + (x+30)^2} = x + 60$$
$$x^2 + (x+30)^2 = (x+60)^2$$
$$x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$$
$$x^2 - 60x - 2700 = 0$$

Question 7: The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Solution:

Let the larger number = x
smaller number = y
As per question,

$$x^2 - y^2 = 180 \qquad ...(1)$$
$$y^2 = 8x \qquad ...(2)$$

Now,

$$x^2 - 8x = 180$$
$$x^2 - 8x - 180 = 0$$
$$x^2 - 18x + 10x - 180 = 0$$
$$x(x-18) + 10(x-18) = 0$$
$$(x-18)(x+10) = 0$$
$$x = 18, -10$$

Hence larger number is 18

$$y^2 = 8 \times 18$$
$$y = \pm 12$$

Smaller number $= \pm 12$.

Question 8: A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution:

Let speed of train = x km/hr
Distance $= 360$ km
Time $= \dfrac{360}{x} hr$
Now,

$$(x+5)\left(\frac{360}{x} - 1\right) = 360$$
$$(x+5)(360-x) = 360x$$
$$360x - x^2 + 1800 - 5x - 360x = 0$$
$$-x^2 - 5x + 1800 = 0$$

$$x^2 + 5x - 1800 = 0$$
$$x^2 + 45x - 40x - 1800 = 0$$
$$x(x+45) - 40(x+45) = 0$$
$$(x-40)(x+45) = 0$$
$$x = 40, -45$$

Speed of train = 40 km/hr

Question 9: Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution:

Let the time taken by the smaller pipe to fill the tank = x hr.

Time taken by the larger pipe = $(x - 10)$ hr

Part of tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of tank filled by larger pipe in 1 hour = $\frac{1}{x-10}$

As per the question,

$$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$
$$75\left(x - 10 + x\right) = 8x\left(x-10\right)$$
$$150x - 750 = 8x^2 - 80x$$
$$8x^2 - 230x + 750 = 0$$
$$8x^2 - 200x - 30x + 750 = 0$$
$$8x(x-25) - 30(x-25) = 0$$
$$(x-25)(8x-30) = 0$$
$$x = 25, \frac{15}{4}$$

$x < 10$ is not possible. Thus, $x = 25$

Hence, the time taken by the smaller pipe to fill the tank = 25 hr.

Time taken by the larger pipe = 15 hr

Question 10: An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speeds of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Solution:

Let the average speed of passenger train be x km/h.

Average speed of express train = $(x + 11)$ km/h

Now,

$$\frac{132}{x} - \frac{132}{x+11} = 1$$
$$132\left(x+11-x\right) = x\left(x+11\right)$$
$$1452 = x^2 + 11x$$
$$x^2 + 11x - 1452 = 0$$

$$x^2 + 44x - 33x - 1452 = 0$$
$$x(x+44) - 33(x+44) = 0$$

$$(x+44)(x-33) = 0$$
$$x = -44, 33$$

Speed of passenger train = 33km/hr

Speed of express train = 44km/hr

Question 11: Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Solution:

Let the sides of the two squares be x m and y m.

Therefore, their perimeter will be $4x$ and $4y$respectively

It is given that

$4x - 4y = 24$

$x - y = 6$

$x = y + 6$

Now,

$$x^2 + y^2 = 468$$
$$(6 + y)^2 + y^2 = 468$$
$$y^2 + 36 + 12y + y^2 - 468 = 0$$
$$2y^2 + 12y - 432 = 0$$
$$y^2 + 6y - 216 = 0$$
$$y^2 + 18y - 12y - 216 = 0$$
$$y(y + 18) - 12(y + 18) = 0$$
$$(y + 18)(y - 12) = 0$$
$$y = 12, -18$$

Sides of the squares are 12m and 18 m.

Exercise: 4.4

Question 1: Find the nature of the roots of the following quadratic equations. If the real roots exist, find them;

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Solution:

We know that for a quadratic equation $ax^2 + bx + c = 0$,

(A) If $b^2 - 4ac > 0 \rightarrow$ two distinct real roots

(B) If $b^2 - 4ac = 0 \rightarrow$ two equal real roots

(C) If $b^2 - 4ac < 0 \rightarrow$ no real roots

(i) $2x^2 - 3x + 5 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we got

$a = 2, b = -3, c = 5$

$d = b^2 - 4ac =$

$d = (-3)^2 - 4(2)(5) = 9 - 40$

$d = -31$

As $b^2 - 4ac < 0$,

Therefore, no real root is possible for the given equation.

(ii) $3x^2 - 4\sqrt{3}x = 4 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we get

$a = 3, b = -4\sqrt{3}, c = 4$

$d = b^2 - 4ac$

$d = 48 - 48$

$d = 0$

As $b^2 - 4ac = 0$,

Therefore, real roots exist for the given equation and they are equal to each other.

Therefore, the roots are $\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$.

(iii) $2x^2 - 6x + 3 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = -6, c = 3$

$d = b^2 - 4ac$

$d = 36 - 24$

$d = 12 > 0$

Therefore, distinct real roots exist

$$x = \frac{6 \pm \sqrt{36-24}}{4}$$
$$x = \frac{6 \pm \sqrt{12}}{4}$$
$$x = \frac{3 \pm \sqrt{3}}{2}$$
$$x = \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$$

Question 2: Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x-2) + 6 = 0$

Solution: We know that if an equation $ax^2 + bx + c = 0$ has two equal roots, its discriminant ($b^2 - 4ac$) will be 0.

(i) $2x^2 + kx + 3 = 0$

Comparing equation with $ax^2 + bx + c = 0$, we obtain

$$a = 2, b = k, c = 3$$

$$d = b^2 - 4ac$$
$$d = k^2 - 24$$

For equal roots,

$$d = 0$$
$$k^2 - 24$$
$$k = \pm 2\sqrt{6}$$

(ii) $kx(x-2) + 6 = 0$

$$kx^2 - 2kx + 6 = 0$$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$$a = k, b = -2k, c = 6$$

$$d = b^2 - 4ac$$
$$d = 4k^2 - 24k$$

For equal roots,

$$d = 0$$
$$4k^2 - 24k = 0$$
$$4k(k - 6) = 0$$
$$k = 0, 6$$

Therefore, if this equation has two equal roots, *k* should be 6 only.

Question 3: Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2?

If so, find its length and breadth.

Solution:

Let the breadth of mango grove be *l*.

Length of mango grove will be 2*l*.

Area of mango grove = (2*l*) (*l*) = 2l^2

$$2l^2 = 800$$
$$l^2 - 400 = 0$$

Comparing this equation with $al^2 + bl + c = 0$, we obtain

$a = 1$ $b = 0$, $c = 400$

$$d = b^2 - 4ac$$
$$d = 0 + 1600$$
$$d = 1600$$

Here, $b^2 - 4ac > 0$

Therefore, the equation will have real roots. And hence, the desired rectangular mango grove can be designed.

$$l = \pm 20$$

However, length cannot be negative.

Therefore, breadth of mango grove = 20 m

Length of mango grove = 2 × 20 = 40 m

Question 4: Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Solution: Let the age of one friend be x years.

Age of the other friend will be $(20 - x)$ years.

4 years ago, age of 1st friend = $(x - 4)$ years

And, age of 2nd friend = $(20 - x - 4) = (16 - x)$ years

Now,

$$(x - 4)(16 - x) = 48$$

$$16x - 64 - x^2 + 4x = 48$$

$$-x^2 + 20x - 112 = 0$$

$$x^2 - 20x + 112 = 0$$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$$a = 1, b = -20, c = 112$$

$$d = b^2 - 4ac = (-20)^2 - 4(1)(112)$$

$$d = 400 - 448 = -48$$

As $b^2 - 4ac < 0$,

Therefore, no real root is possible for this equation and hence, this situation is not possible.

Question 5: Is it possible to design a rectangular park of perimeter 80 and area 400 m^2? If so find its length and breadth.

Solution: Let the length and breadth of the park be l and b.

Perimeter = $2(l + b) = 80$

$$l + b = 40$$

or, $b = 40 - l$

Area $= l \times b = l\,(40 - l) = 40l - l^2$

$40l - l^2 = 400$

$l^2 - 40l + 400 = 0$

Comparing this equation with

$al^2 + bl + c = 0$, we obtain

$a = 1, b = -40, c = 400$

Discriminate $= b^2 - 4ac = (-40)^2 - 4\,(1)\,(400)$

$= 1600 - 1600 = 0$

As $b^2 - 4ac = 0$,

Therefore, this equation has equal real roots. And hence, this situation is possible.

Root of this equation,

$l = 20$

Therefore, length of park, $l = 20$ m

And breadth of park, $b = 40 - l = 40 - 20 = 20$ m