Chapter 1
Real Numbers

Exercise: 1.1

Question 1: Use Euclid's division algorithm to find the HCF of:

Solution:

(i) 135 and 225

$225 > 135$,

$225 = 135 \times 1 + 90$

Also,

$135 = 90 \times 1 + 45$

$90 = 2 \times 45 + 0$

Therefore, the HCF of 135 and 225 is 45.

(ii) 196 and 38220

$38220 > 196$,

$38220 = 196 \times 195 + 0$

Therefore, HCF of 196 and 38220 is 196.

(iii) 867 and 255

Since $867 > 255$,

$867 = 255 \times 3 + 102$

$255 = 102 \times 2 + 51$

$$102 = 51 \times 2 + 0$$

Therefore, HCF of 867 and 255 is 51.

Question 2: Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Solution:

By Euclid's algorithm,

$$a = 6q + r, \text{ and } r = 0, 1, 2, 3, 4, 5$$

Hence, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

`Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where k is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2.

Hence, these numbers are odd numbers.

Question 3: An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution:

Euclid's algorithm

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0$$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

Question 4: Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m

Solution:

$a = 3q + r$, $r = 0, 1, 2$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

Question 5: Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$

Solution:

$a = 3q + r,\ 0 \le r < 3$

$$a = 3q, 3q + 1, 3q + 2$$

There are three cases.

Case 1: When $a = 3q$,

$$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$$

Case 2: When $a = 3q + 1$,

$a^3 = (3q + 1)^3$

$a^3 = 27q^3 + 27q^2 + 9q + 1$

$a^3 = 9(3q^3 + 3q^2 + q) + 1$

$a^3 = 9m + 1$

Case 3: When $a = 3q + 2$,

$a^3 = (3q + 2)^3$

$a^3 = 27q^3 + 54q^2 + 36q + 8$

$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$

$a^3 = 9m + 8$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

Exercise: 1.2

Question 1: Express each number as a product of its prime factors:

Solution:

$(i)\ 140 = 2^2 \times 5 \times 7$

$(ii)\ 156 = 2^3 \times 7$

$(iii)\ 3825 = 3^2 \times 5^2 \times 17$

$(iv)\ 5005 = 5 \times 7 \times 11 \times 13$

$(v)\ 7429 = 17 \times 19 \times 23$

Question 2: Find the LCM and HCF of the following pairs of integers and veriry that LCM x HCF = product of the two numbers.

Solution:

(i) 26 and 91

$26 = 2 \times 13$

$91 = 7 \times 13$

$HCF = 13$

$LCM = 14 \times 13 = 182$

Product of two numbers $= 26 \times 91 = 2366$

$HCF \times LCM = 13 \times 182 = 2366$

Hence, it is verified.

(ii) 510 and 92

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

$HCF = 2$

$LCM = 4 \times 15 \times 17 \times 23 = 23460$

Product of two numbers $= 510 \times 92 = 46920$

$HCF \times LCM = 2 \times 23460 = 46920$

Hence, it is verified.

(iii) 336 and 54

$$336 = 2\times2\times2\times2\times3\times7$$
$$54 = 2\times3\times3\times3$$
$$HCF = 6$$
$$LCM = 8\times81\times7 = 3024$$
$$\text{Product of two numbers} = 336\times54 = 18144$$
$$HCF\times LCM = 6\times3024 = 18144$$

Hence, it is verified.

Question 3: Find the LCM and HCF of the following integers by applying the prime factorisation method.

Solution:

(i) 12,15 and 21

$$12 = 2\times2\times3$$
$$15 = 3\times5$$
$$21 = 3\times7$$
$$HCF = 3$$
$$LCM = 4\times15\times7 = 420$$

(ii) 17,23 and 29

$$17 = 17\times1$$
$$23 = 23\times1$$
$$29 = 29\times1$$
$$HCF = 1$$
$$LCM = 17\times23\times29 = 11339$$

(iii) 8,9 and 25

$$8 = 2\times2\times2$$
$$9 = 3\times3$$
$$25 = 5\times5$$
$$HCF = 1$$
$$LCM = 8\times9\times25 = 1800$$

Question 4: Given that HCF (306, 657) = 9, find LCM (306, 657).

Solution:

$$306 = 2\times3\times3\times17$$
$$657 = 3\times3\times73$$
$$HCF = 9$$
$$LCM = 9\times34\times73 = 22338$$

Question 5: Check whether 6^n can end with the digit 0 for any natural number n

Solution:

Prime factorisation of 6^n = (2 x3)"
It can be observed that 5 is not in the prime factorisation of 6^n
Hence, for any value of *n*, 6^n will not be divisible by 5.

Exercise: 1.3

Question 1: Prove that $\sqrt{5}$ is irrational

Solution:

$$Let\ a = 5k$$

$$(5k)^2 = 5b^2$$
$$b^2 = 5b^2$$

Thus, a and b have 5 as a common factor.

Hence, $\sqrt{5}$ is a irrational no.

Question 2: Prove that $3+2\sqrt{5}$ is irrational

Solution:

Let $3+2\sqrt{5}$ is a rational number.

Thus,

$$3+2\sqrt{5}=\frac{a}{b}$$

$$\sqrt{5}=\frac{1}{2}\left(\frac{a}{b}-3\right)$$

$\frac{1}{2}\left(\frac{a}{b}-3\right)$ is rational

Hence, $3+2\sqrt{5}$ is a irrational no.

Question 3: Prove that the following are irrational:

Solution:

(i) $\frac{1}{\sqrt{2}}$

Let $\frac{1}{\sqrt{2}}$ irrational number.

Thus,

$$\frac{1}{\sqrt{2}}=\frac{a}{b}$$

$$\sqrt{2} = \frac{b}{a}$$

$\frac{b}{a}$ is rational

Hence, $\frac{1}{\sqrt{2}}$ is a irrational no.

(ii) $7\sqrt{5}$

Let $7\sqrt{5}$ irrational number.

Thus,

$$7\sqrt{5} = \frac{a}{b}$$

$$\sqrt{5} = \frac{a}{7b}$$

$\frac{a}{7b}$ is rational

Hence, $7\sqrt{5}$ is a irrational no.

(iii) $6+\sqrt{2}$

Let $6+\sqrt{2}$ irrational number.

Thus,

$$6+\sqrt{2} = \frac{a}{b}$$

$$\sqrt{2} = \frac{a}{b} - 6$$

$\left(\frac{a}{b} - 6\right)$ is rational

Hence, $6+\sqrt{2}$ is a irrational no.

Exercise: 1.4

Question 1: Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

Solution:

(i)

$$\frac{13}{3125}$$

$3125 = 5^5$

The denominator is in the form 5^n.

Thus, decimal expression is terminating

(ii)

$$\frac{17}{8}$$

$8 = 2^3$

The denominator is in the form 2^n.

Thus, decimal expression is terminating

(iii)

$$\frac{64}{455}$$

$455 = 5 \times 7 \times 13$

Thus, decimal expression is non-terminating

(iv)

$$\frac{15}{1600}$$

$1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

The denominator is in the form $2^n \times 5^n$.

Thus, decimal expression is terminating

(v)

$$\frac{29}{343}$$

$343 = 7 \times 7 \times 7$

The denominator is in the form 7^n.

Thus, decimal expression is non-terminating repeating

(vi)

$$\frac{23}{2^3 \times 5^2}$$

The denominator is in the form $2^n \times 5^n$

Thus, decimal expression is terminating

(vii)

$$\frac{129}{2^2 \times 5^7 \times 7^5}$$

The denominator is in the form $2^n \times 5^m \times 7^p$

Thus, decimal expression is non-terminating repeating

(viii)

$$\frac{6}{15} = \frac{2}{5}$$

The denominator is in the form 5^n

Thus, decimal expression is terminating

(ix)

$$\frac{35}{50} = \frac{7}{10}$$

The denominator is in the form $2^m \times 5^n$

Thus, decimal expression is terminating

(x)

$$\frac{77}{210} = \frac{11}{30}$$
$$30 = 2 \times 3 \times 5$$

The denominator is in the form $2^m \times 5^n \times 3^p$

Thus, decimal expression is non-terminating repeating

Question 2: Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Solution:

(i) $\frac{13}{3125} = 0.00416$

(ii) $\frac{17}{8} = 2.125$

(iv) $\frac{15}{1600} = 0.009375$

(vi) $\frac{23}{200} = 0.115$

(viii) $\frac{6}{15} = 0.4$

(ix) $\frac{35}{50} = 0.7$

Question 3: The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not, If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factor of g?

Solution:

(i)

43.123456789

q is the form of $2^m \times 5^n$

(ii)

0.120120012000......

The decimal expresion is neither terminating nor recurring

(iii)

$43.\overline{123456789}$

The decimal expression is non terminating recuring