

## Chapter 10

### Circles

#### Exercise: 10.1

**Question 1:** How many tangents can a circle have?

**Solution:** A circle can have infinite tangents.

**Question 2:** Fill in the blanks:

A tangent to a circle intersects it in \_\_\_\_\_ point (s).

A line intersecting a circle in two points is called a \_\_\_\_\_.

A circle can have \_\_\_\_\_ parallel tangents at the most.

The common point of a tangent to a circle and the circle is called \_\_\_\_\_.

**Solution:**

One

Secant

Two

Point of contact

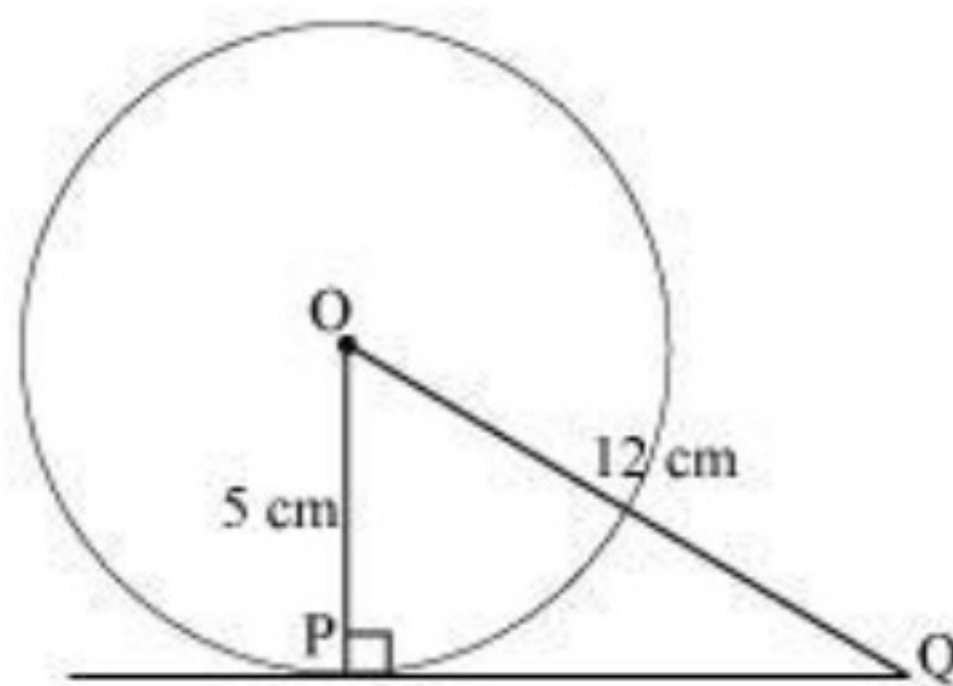
**Question 3:** A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :

(A) 12 cm. (B) 13 cm (C) 8.5 cm (D)  $\sqrt{119}$  cm

**Solution:** We know that the line drawn from the centre of the circle to the tangent is perpendicular to the tangent.

$$\therefore OP \perp PQ$$

By applying Pythagoras theorem in  $\triangle OPQ$ ,



$$\therefore OP^2 + PQ^2 = OQ^2$$

$$5^2 + PQ^2 = 12^2$$

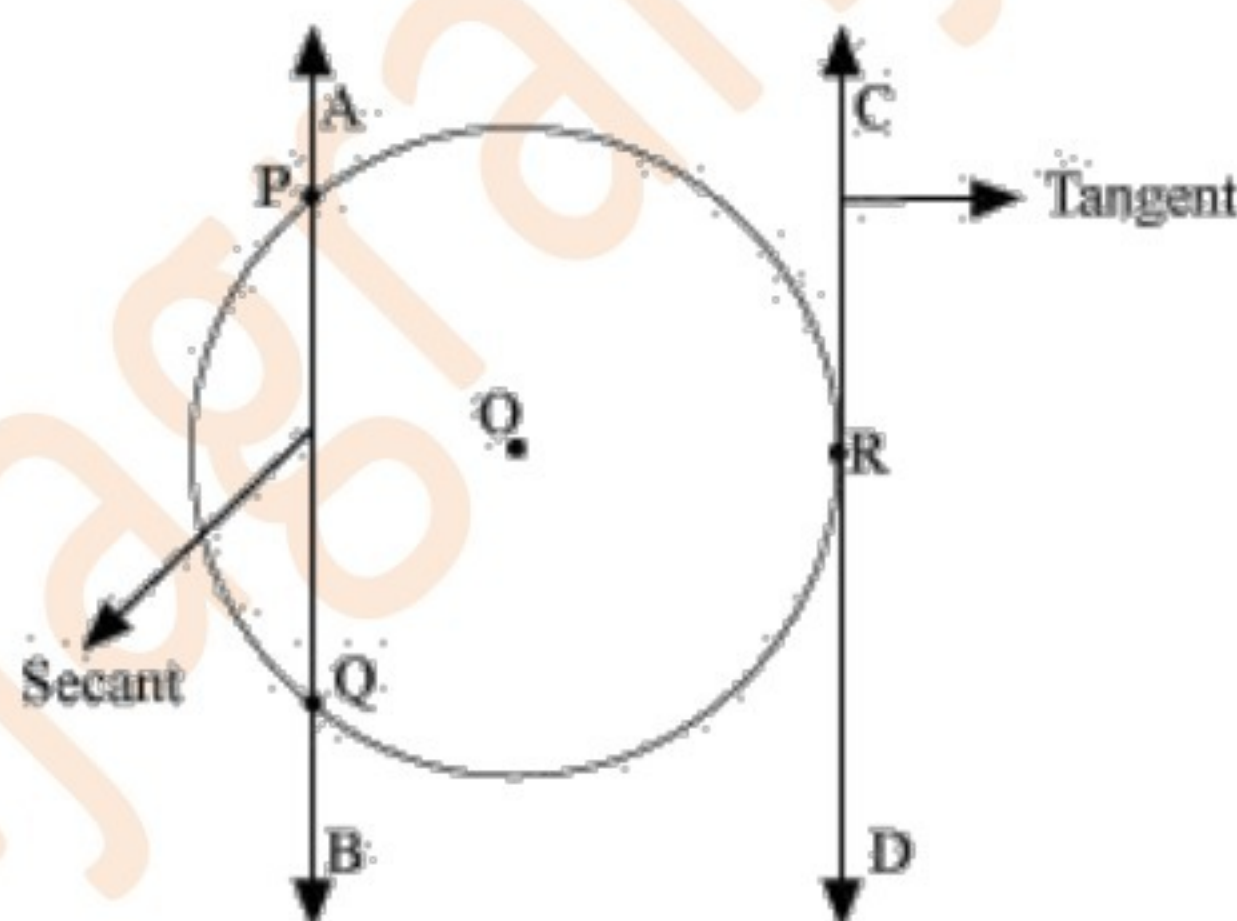
$$PQ^2 = 144 - 25$$

$$PQ = \sqrt{119} \text{ cm.}$$

Hence, the correct answer is (D).

**Question 4:** Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

**Solution:**



$AB \parallel CD$

Line AB is intersecting the circle at exactly two points, P and Q.

Therefore, line AB is the secant of this circle.

Since line CD is intersecting the circle at exactly one point, R, line CD is the tangent to the circle.

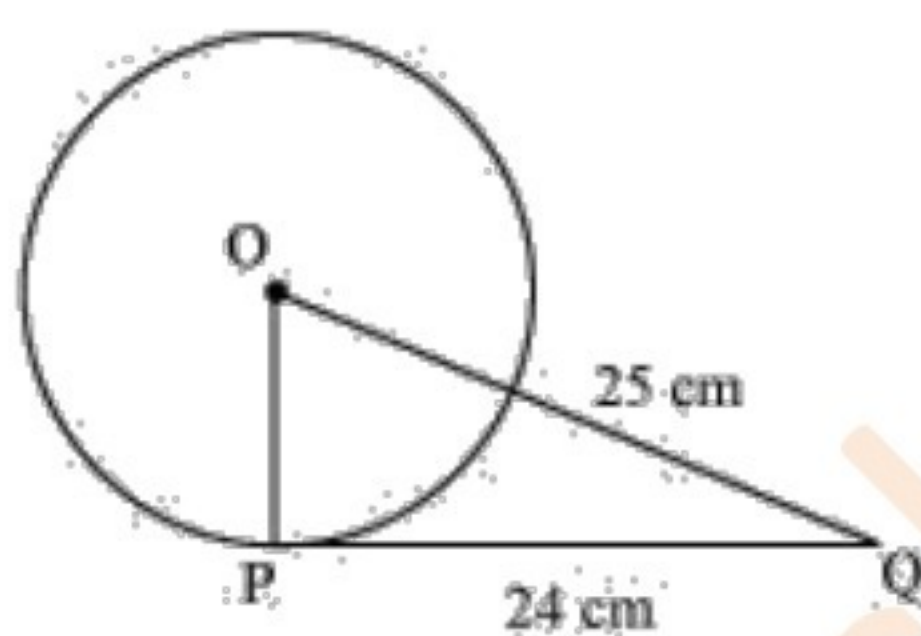


## Exercise: 10.2

**Question 1:** From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(A) 7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm

**Solution:**



Let O be the centre of the circle.

Given that,

$OQ = 25\text{ cm}$  and  $PQ = 24\text{ cm}$

$OP \perp PQ$  (radius is perpendicular to the tangent at the point of contact)

Applying Pythagoras theorem in  $\triangle OPQ$ ,

$$OP^2 + PQ^2 = OQ^2$$

$$OP^2 + 24^2 = 25^2$$

$$OP^2 = 625 - 576$$

$$OP^2 = 49$$

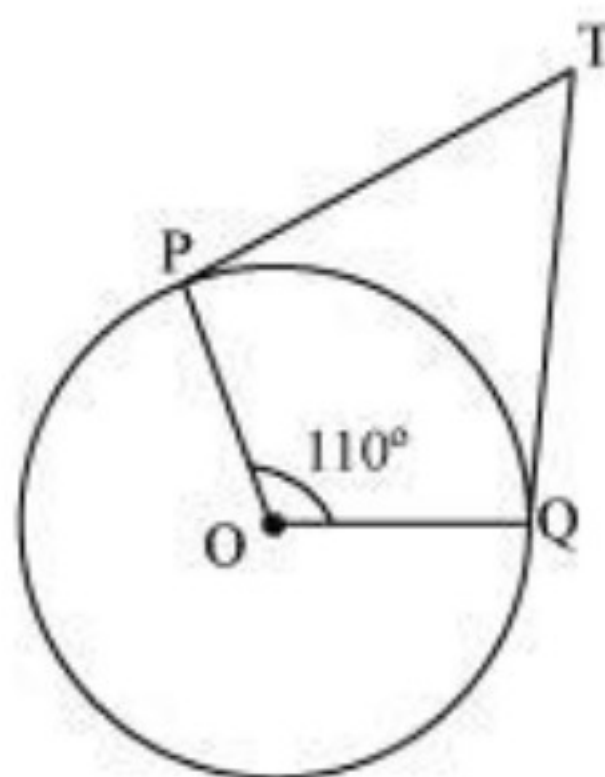
$$OP = 7$$

Therefore, the radius of the circle is 7 cm.

Hence, alternative (A) is correct.

**Question 2:** In the given figure, if TP and TQ are the two tangents to a circle with centre O so that  $\angle POQ = 110^\circ$ , then  $\angle PTQ$  is equal to

(A)  $60^\circ$  (B)  $70^\circ$  (C)  $80^\circ$  (D)  $90^\circ$



**Solution:** It is given that TP and TQ are tangents.

Therefore, radius drawn to these tangents will be perpendicular to the tangents.

Thus,  $OP \perp TP$  and  $OQ \perp TQ$

$$\angle OPT = 90^\circ$$

$$\angle OQT = 90^\circ$$

In quadrilateral POQT,

$$\text{Sum of all interior angles} = 360^\circ$$

$$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360^\circ$$

$$\Rightarrow 90^\circ + 110^\circ + 90^\circ + \angle PTQ = 360^\circ$$

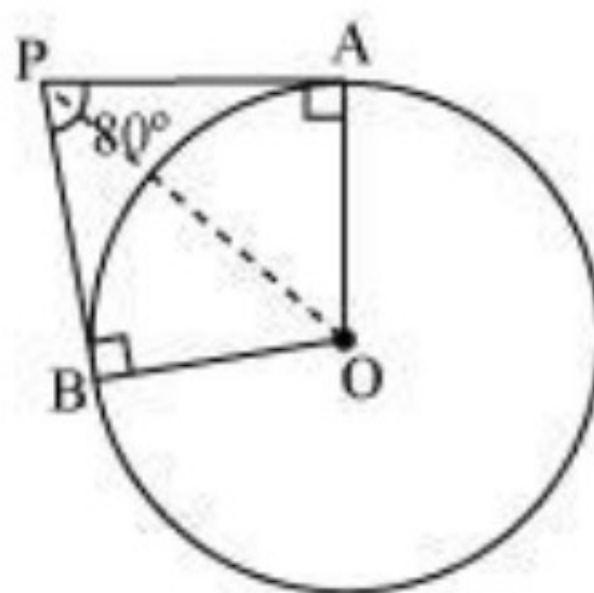
$$\Rightarrow \angle PTQ = 70^\circ$$

Hence, alternative (B) is correct.

**Question 3:** If tangents PA and PB from a point P to a circle with centre O are inclined to each other an angle of  $80^\circ$ , then  $\angle POA$  is equal to

(A)  $50^\circ$  (B)  $60^\circ$  (C)  $70^\circ$  (D)  $80^\circ$

**Solution:** It is given that PA and PB are tangents.



Therefore, the radius drawn to these tangents will be perpendicular to the tangents.



Thus,  $OA \perp PA$  and  $OB \perp PB$

$$\angle OBP = 90^\circ$$

$$\angle OAP = 90^\circ$$

In AOBP,

Sum of all interior angles =  $360^\circ$

$$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$$

$$90^\circ + 80^\circ + 90^\circ + \angle BOA = 360^\circ$$

$$\angle BOA = 100^\circ$$

In  $\triangle OPB$  and  $\triangle OPA$ ,

$AP = BP$  (Tangents from a point)

$OA = OB$  (Radii of the circle)

$OP = OP$  (Common side)

Therefore,  $\triangle OPB \cong \triangle OPA$  (SSS congruence criterion)

$A \leftrightarrow B, P \leftrightarrow P, O \leftrightarrow O$

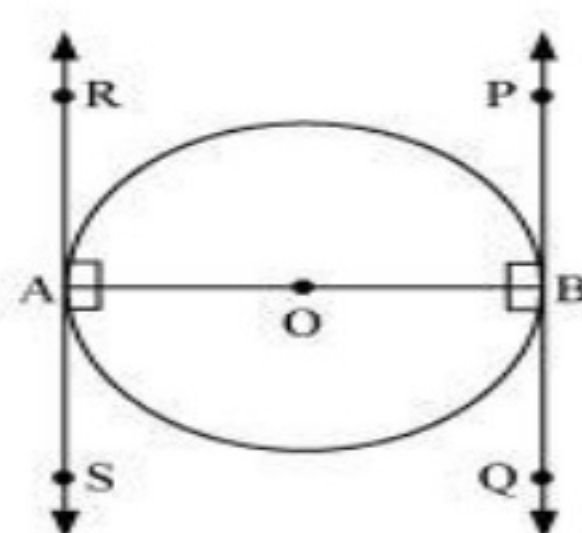
And thus,  $\angle POB = \angle POA$

$$\angle POA = \frac{1}{2} \angle AOB = \frac{100^\circ}{2} = 50^\circ$$

Hence, alternative (A) is correct.

**Question 4:** Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

**Solution:**



Let AB be a diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively.

Thus,  $OA \perp RS$  and  $OB \perp PQ$

$$\angle OAR = 90^\circ$$

$$\angle OAS = 90^\circ$$

$$\angle OBP = 90^\circ$$

$$\angle OBQ = 90^\circ$$

It can be observed that

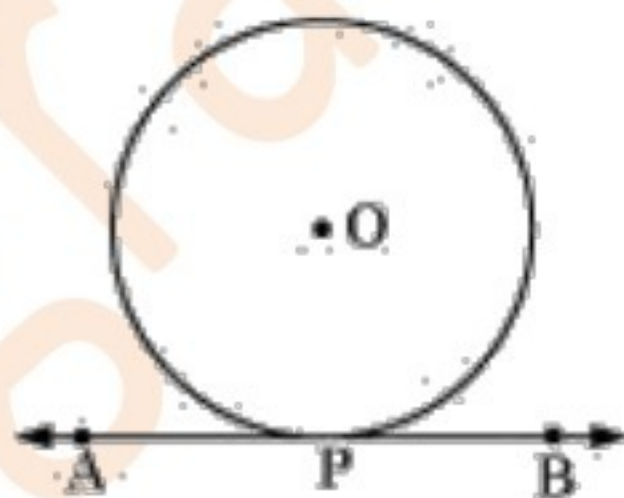
$$\angle OAR = \angle OBQ \text{ (Alternate interior angles)}$$

$$\angle OAS = \angle OBP \text{ (Alternate interior angles)}$$

Since alternate interior angles are equal, lines PQ and RS will be parallel.

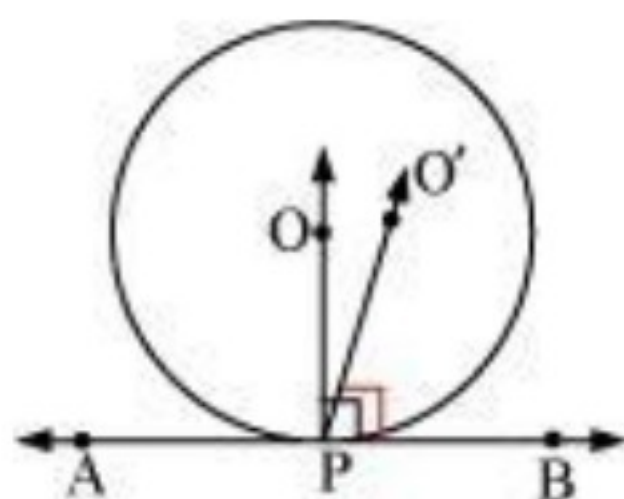
**Question 5:** Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

**Solution:** Let us consider a circle with centre O. Let AB be a tangent which touches the circle at P.



Let us assume that the perpendicular to AB at P does not pass through centre O. Let it pass through another point O'.

Join OP and O'P



As perpendicular to AB at P passes through O', therefore,

$$\angle O'PB = 90^\circ \dots (1)$$

Line joining the centre and the point of contact to the tangent of the circle are perpendicular to each other.

**Get Practice Papers, Solved Question Papers, Syllabus, Sample Papers, Expert's video, Online Test and much more....**



$$\therefore \angle OPB = 90^\circ \dots (2)$$

Comparing equations (1) and (2), we obtain

$$\angle O'PB = \angle OPB \dots (3)$$

From the figure, it can be observed that,

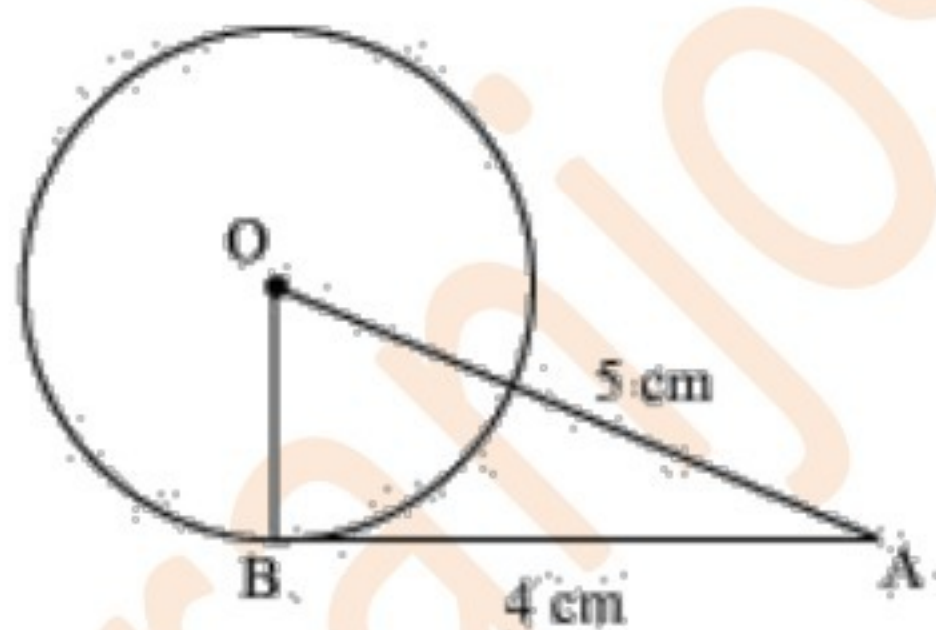
$$\angle O'PB < \angle OPB \dots (4)$$

Therefore,  $\angle O'PB = \angle OPB$  is not possible. It is only possible, when the line  $O'P$  coincides with  $OP$ .

Therefore, the perpendicular to  $AB$  through  $P$  passes through centre  $O$ .

**Question 6:** The length of a tangent from a point  $A$  at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

**Solution:**



Let us consider a circle centred at point  $O$ .

$AB$  is a tangent drawn on this circle from point  $A$ .

Given that,

$$OA = 5\text{ cm and } AB = 4\text{ cm}$$

In  $\triangle ABO$ ,

$OB \perp AB$  (Radius  $\perp$  tangent at the point of contact)

Applying Pythagoras theorem in  $\triangle ABO$ ,

$$AB^2 + BO^2 = OA^2$$

$$4^2 + BO^2 = 5^2$$

$$16 + BO^2 = 25$$

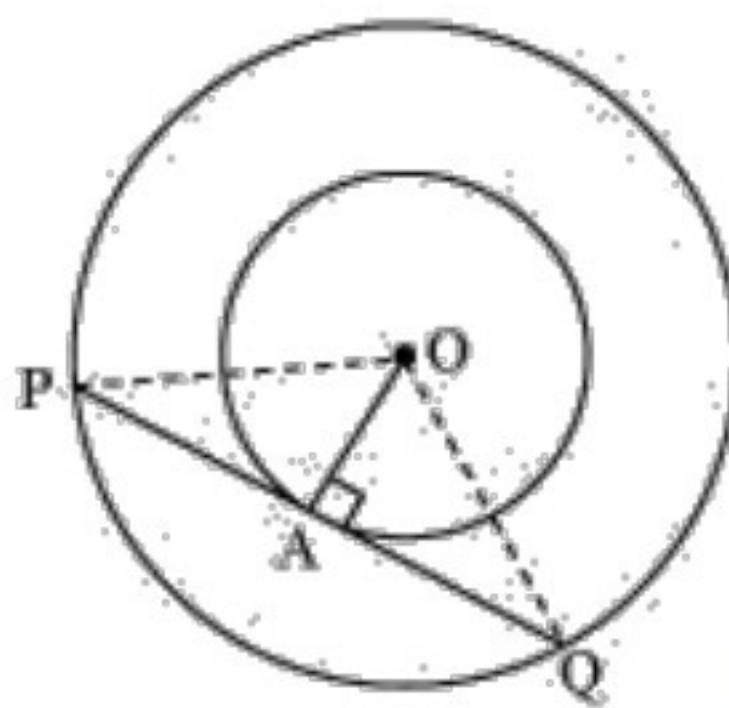
$$BO^2 = 9$$

$$BO = 3$$

Hence, the radius of the circle is 3 cm.

**Question 7:** Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

**Solution:**



Let the two concentric circles be centred at point O. And let PQ be the chord of the larger circle which touches the smaller circle at point A. Therefore, PQ is tangent to the smaller circle.

$OA \perp PQ$  (As OA is the radius of the circle)

Applying Pythagoras theorem in  $\triangle OAP$ , we obtain

$$OA^2 + AP^2 = OP^2$$

$$3^2 + AP^2 = 5^2$$

$$9 + AP^2 = 25$$

$$AP^2 = 16$$

$$AP = 4$$

In  $\triangle OPQ$ ,

Since  $OA \perp PQ$ ,

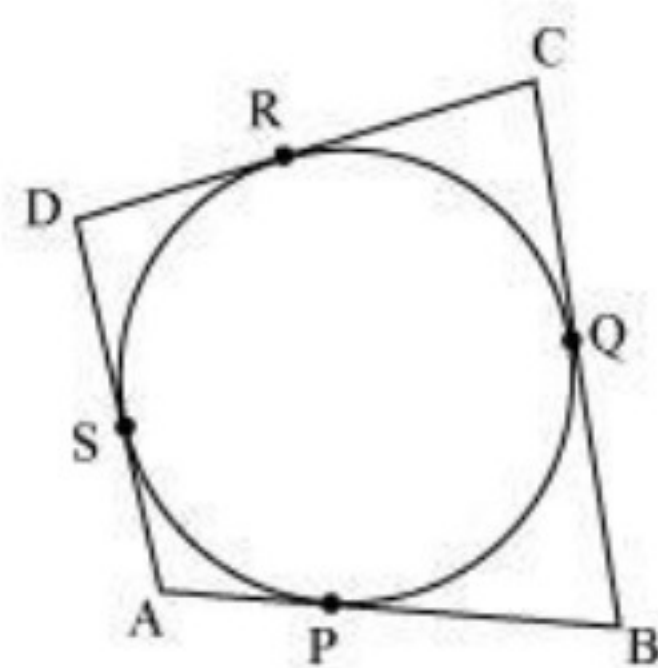
$AP = AQ$  (Perpendicular from the centre of the circle bisects the chord)

$$\text{Thus, } PQ = 2AP = 2 \times 4 = 8$$

Therefore, the length of the chord of the larger circle is 8 cm.

**Question 8:** A quadrilateral ABCD is drawn to circumscribe a circle (see given figure) Prove that  $AB + CD = AD + BC$





**Solution:** It can be observed that

$$DR = DS \text{ (Tangents on the circle from point D) ... (1)}$$

$$CR = CQ \text{ (Tangents on the circle from point C) ... (2)}$$

$$BP = BQ \text{ (Tangents on the circle from point B) ... (3)}$$

$$AP = AS \text{ (Tangents on the circle from point A) ... (4)}$$

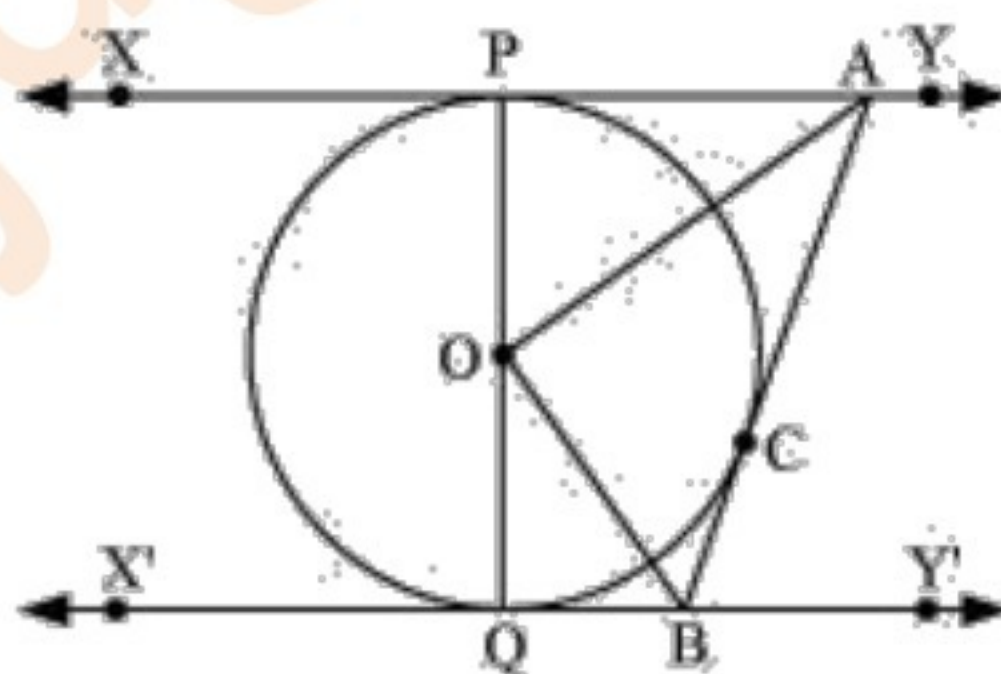
Adding all these equations, we obtain

$$DR + CR + BP + AP = DS + CQ + BQ + AS$$

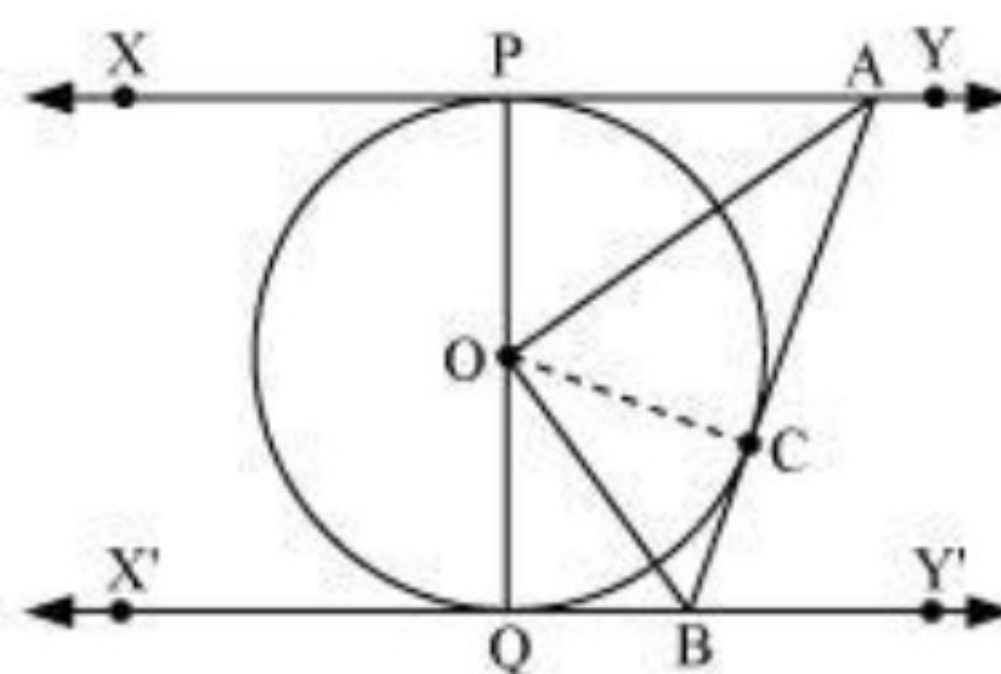
$$(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$$

$$CD + AB = AD + BC$$

**Question 9:** In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that  $\angle AOB = 90^\circ$ .



**Solution:** Let us join point O to C.



In  $\triangle OPA$  and  $\triangle OCA$ ,

$OP = OC$  (Radii of the same circle)

$AP = AC$  (Tangents from point A)

$AO = AO$  (Common side)

$\triangle OPA \cong \triangle OCA$  (SSS congruence criterion)

Therefore,  $P \leftrightarrow C$ ,  $A \leftrightarrow A$ ,  $O \leftrightarrow O$

$\angle POA = \angle COA \dots (i)$

Similarly,  $\triangle OQB \cong \triangle OCB$

$\angle QOB = \angle COB \dots (ii)$

Since  $POQ$  is a diameter of the circle, it is a straight line.

Therefore,  $\angle POA + \angle COA + \angle COB + \angle QOB = 180^\circ$

From equations (i) and (ii), it can be observed that

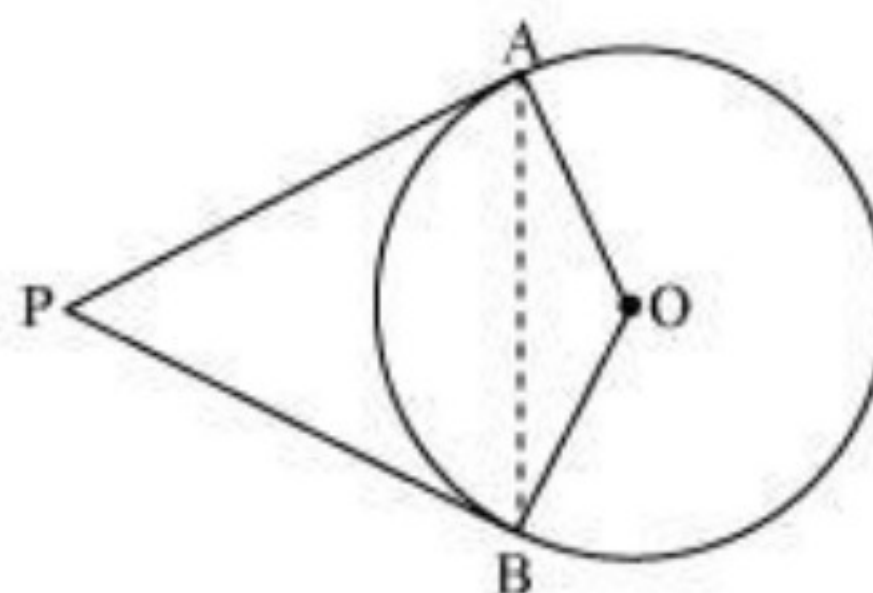
$$2\angle COA + 2\angle COB = 180^\circ$$

$$\angle COA + \angle COB = 90^\circ$$

$$\angle AOB = 90^\circ$$

**Question 10:** Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

**Solution:**



Let us consider a circle centred at point O. Let P be an external point from which two tangents PA and PB are drawn to the circle which are touching the circle at point A and B respectively and AB is the line segment, joining point of contacts A and B together such that it subtends  $\angle AOB$  at centre O of the circle.

$OA$  (radius)  $\perp$   $PA$  (tangent)



Therefore,  $\angle OAP = 90^\circ$

Similarly,  $OB$  (radius)  $\perp$   $PB$  (tangent)

$$\angle OBP = 90^\circ$$

In quadrilateral  $OAPB$ ,

Sum of all interior angles =  $360^\circ$

$$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$$

$$90^\circ + \angle APB + 90^\circ + \angle BOA = 360^\circ$$

$$\angle APB + \angle BOA = 180^\circ$$

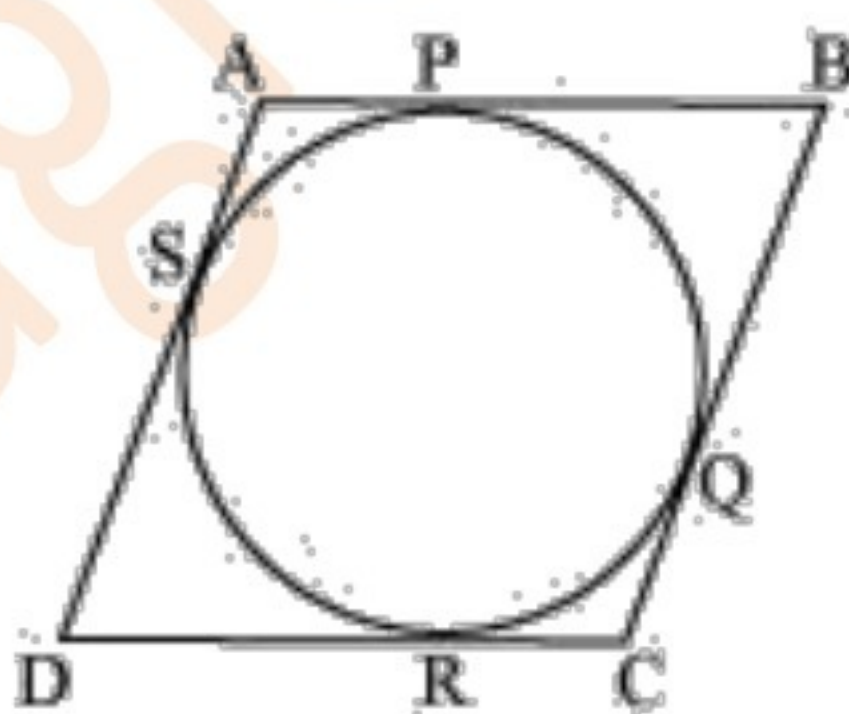
Hence, it can be observed that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

**Question 11:** Prove that the parallelogram circumscribing a circle is a rhombus.

**Solution:** Since  $ABCD$  is a parallelogram,

$$AB = CD \dots (1)$$

$$BC = AD \dots (2)$$



$DR = DS$  (Tangents on the circle from point  $D$ )

$CR = CQ$  (Tangents on the circle from point  $C$ )

$BP = BQ$  (Tangents on the circle from point  $B$ )

$AP = AS$  (Tangents on the circle from point  $A$ )

Adding all these equations, we obtain

$$DR + CR + BP + AP = DS + CQ + BQ + AS$$

$$(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$$

$$CD + AB = AD + BC$$

On putting the values of equations (1) and (2) in this equation, we obtain

$$2AB = 2BC$$

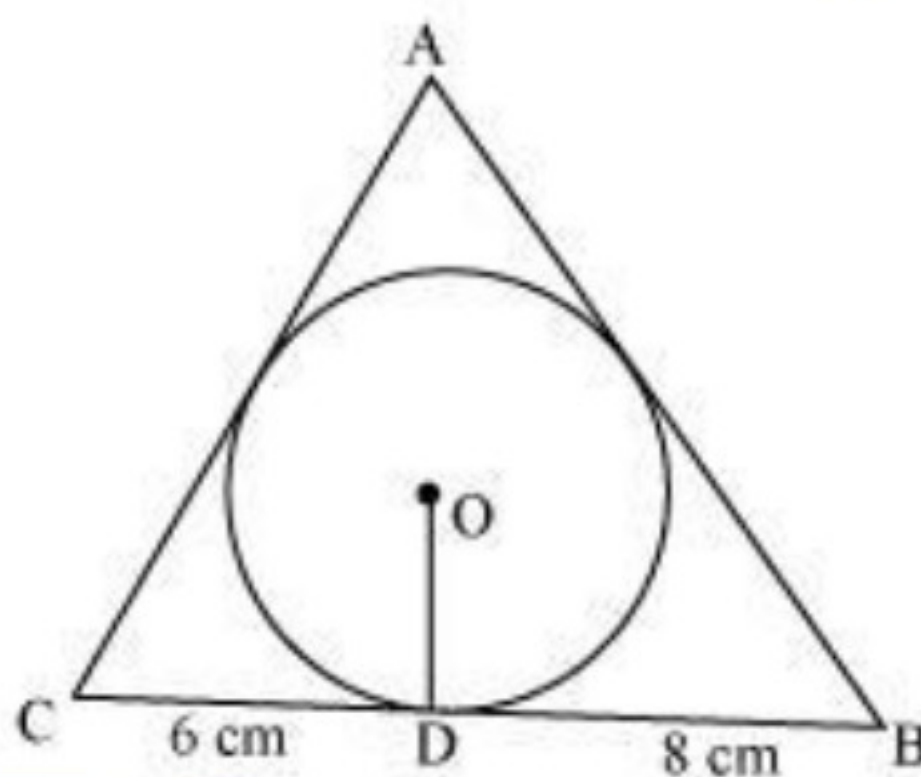
$$AB = BC \dots (3)$$

Comparing equations (1), (2), and (3), we obtain

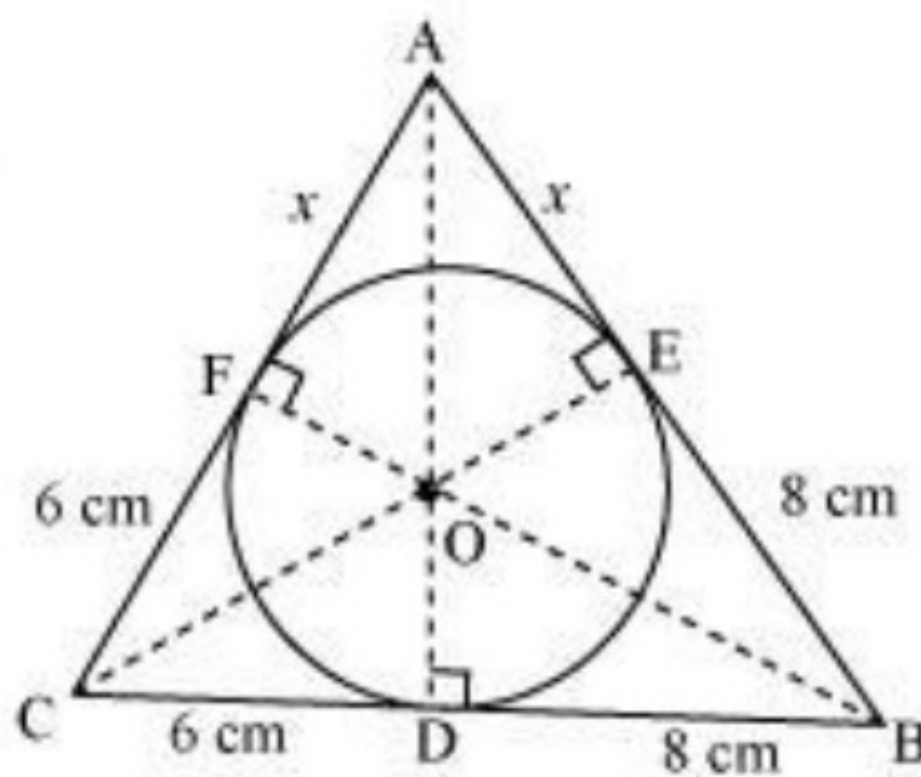
$$AB = BC = CD = DA$$

Hence, ABCD is a rhombus.

**Question 12:** A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see given figure). Find the sides AB and AC.



**Solution:** Let the given circle touch the sides AB and AC of the triangle at point E and F respectively and the length of the line segment AF be  $x$ .



In  $\triangle ABC$ ,

$CF = CD = 6\text{ cm}$  (Tangents on the circle from point C)

$BE = BD = 8\text{ cm}$  (Tangents on the circle from point B)

$AE = AF = x$  (Tangents on the circle from point A)



$$AB = AE + EB = x + 8$$

$$BC = BD + DC = 8 + 6 = 14$$

$$CA = CF + FA = 6 + x$$

$$2s = AB + BC + CA$$

$$= x + 8 + 14 + 6 + x$$

$$= 28 + 2x$$

$$s = 14 + x$$

$$\begin{aligned}\text{Area of } \triangle ABC &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{(14+x)(14+x-14)(14+x-6-x)(14+x-8-x)} \\ &= 4\sqrt{3x(14+x)}\end{aligned}$$

$$\text{Area of } \triangle OBC = \frac{1}{2} \times OD \times BC = \frac{1}{2} \times 4 \times 14 = 28$$

$$\text{Area of } \triangle OCA = \frac{1}{2} \times OF \times AC = \frac{1}{2} \times 4 \times (6+x) = 12 + 2x$$

$$\text{Area of } \triangle OAB = \frac{1}{2} \times OE \times AB = \frac{1}{2} \times 4 \times (8+x) = 16 + 2x$$

$$\text{Area of } \triangle ABC = \text{Area of } \triangle OBC + \text{Area of } \triangle OCA + \text{Area of } \triangle OAB$$

$$4\sqrt{3x(14+x)} = 28 + 12 + 2x + 16 + 2x$$

$$4\sqrt{3x(14+x)} = 56 + 4x$$

$$\sqrt{3x(14+x)} = 14 + x$$

$$3x(14+x) = 196 + x^2 + 28x$$

$$42x + 3x^2 = 196 + x^2 + 28x$$

$$2x^2 + 14x - 196 = 0$$

$$x^2 + 7x - 98 = 0$$

$$x^2 + 14x - 7x - 98 = 0$$

$$x(x+14) - 7(x+14) = 0$$

$$(x+14)(x-7)=0$$
$$x=7, -14$$

However,  $x = -14$  is not possible as the length of the sides will be negative.

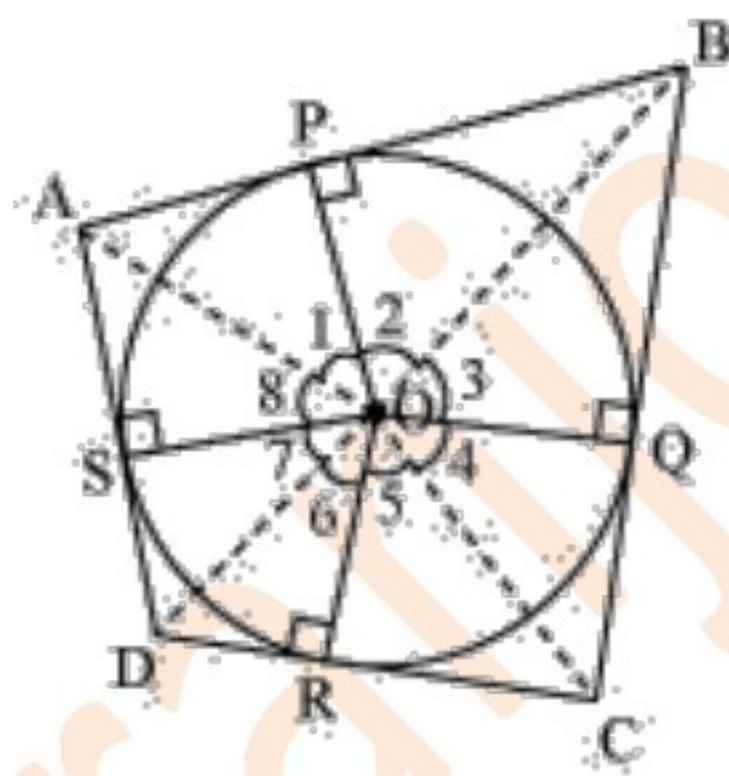
Therefore,  $x = 7$

Hence,  $AB = x + 8 = 7 + 8 = 15$  cm

$CA = 6 + x = 6 + 7 = 13$  cm

**Question 13:** Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

**Solution:**



Let ABCD be a quadrilateral circumscribing a circle centred at O such that it touches the circle at point P, Q, R, S. Let us join the vertices of the quadrilateral ABCD to the centre of the circle.

Consider  $\triangle OAP$  and  $\triangle OAS$ ,

$AP = AS$  (Tangents from the same point)

$OP = OS$  (Radii of the same circle)

$OA = OA$  (Common side)

$\triangle OAP \cong \triangle OAS$  (SSS congruence criterion)

Therefore,  $A \leftrightarrow A, P \leftrightarrow S, O \leftrightarrow O$

And thus,  $\angle POA = \angle AOS$

$$\angle 1 = \angle 8$$

Similarly,

$$\angle 2 = \angle 3$$



$$\angle 4 = \angle 5$$

$$\angle 6 = \angle 7$$

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$

$$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^\circ$$

$$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^\circ$$

$$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^\circ$$

$$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^\circ$$

$$\angle AOB + \angle COD = 180^\circ$$

Similarly, we can prove that  $\angle BOC + \angle DOA = 180^\circ$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.